

# A New Architecture for Transformation-Based Generators<sup>1</sup>

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**Abstract.** A serious problem of many transformation-based generators is that they are trying to achieve three mutually antagonistic goals simultaneously: 1) deeply factored operators and operands to gain the combinatorial programming leverage provided by composition, 2) high performance code in the generated program, and 3) small (i.e., practical) generation search spaces. The hypothesis of this paper is that current generator structures are inadequate to fully achieve these goals because they often induce an explosion of the generation search space. Therefore, new architectures are required. A generator has been implemented in Common LISP to explore architectural variations needed to address this quandary. It is called the Anticipatory Optimization Generator (AOG<sup>3</sup>) because it allows programmers to **anticipate** optimization opportunities and to prepare an abstract, distributed plan that attempts to achieve them. More generally, AOG uses several strategies to prevent generation search spaces from becoming an explosion of choices but the fundamental principle underlying all of them is to solve separate, narrow and specialized generation problems by strategies tailored to each individual problem rather than attempting to solve all problems by a single, general strategy. A second fundamental notion is the preservation and use of domain specific information as a way to gain leverage on generation problems. This paper will focus on two specific mechanisms: 1) *Localization*: The generation and merging of implicit control structures, and 2) *Tag-Directed Transformations*: A new control structure for transformation-based optimization that allows differing kinds of domain knowledge (e.g., optimization knowledge) to be anticipated, affixed to the component parts in the reuse library, and triggered when the time is right for its use.

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# 1 Introduction

## 1.1 The General Problem

A serious problem of most pattern-directed, transformation-based program generators is that they are trying to achieve three mutually antagonistic goals simultaneously: 1) deeply factored and highly abstract operators and operands to gain the combinatorial programming leverage provided by compositions of abstractions, 2) high performance code in the generated program, and 3) small (i.e., practical) generation search spaces. Various program generators focus on one or two of these goals thereby often compromising the other goal(s). This paper will make the argument that this quandary is due in large measure to deficiencies of conventional pattern-directed transformation models. While pattern-directed (PD) transformations (also called *rules*) make the specification of the transformation steps easy and, in fact, generally do a pretty good job of refining (i.e., translating) abstractions into code, they often explode the search space when one is later trying to produce highly optimized code from compositions of deeply factored, abstract operators and operands. Since giving up the deep factoring of abstractions to reduce the search spaces also gives up the combinatorial programming leverage provided by the composition, it is not a good trade-off.

Before addressing the detailed sub-problems and proposing solutions, the next two sections will provide a short introduction to PD transformation systems and discuss the nature of the search space explosion problem.

## 1.2 Pattern-Directed Control Regimes

In the simplest form, generic pattern-directed transformation systems store knowledge as a single global soup of transformations<sup>4</sup> represented as rewrite rules of the form<sup>5</sup>

***Pattern***  $\Rightarrow$  ***RewrittenExpression***

The left hand side of the rule (i.e., ***Pattern***) matches a subtree of an Abstract Syntax Tree (AST) and binds matching elements of that subtree to variables (e.g., **?operator**) in the pattern. If successful, the right hand side (rhs) (i.e., ***RewrittenExpression***), instantiated with the variable bindings, replaces the matched portion of the subtree. Operationally, rules are chosen (i.e., triggered) based largely on the pattern of the left hand side, thereby motivating the moniker “Pattern-Directed” for such systems. Beyond syntactic forms, rules may also include 1) semantic constraints (e.g., type restrictions), and 2) domain constraints, that must be true before the rule can be triggered. Such constraints are called *enabling conditions*. The checking of enabling conditions and other translation chores (e.g., generating translator variables) are often handled by a separate procedure associated with the rule.

One of the key questions with transformation systems is what is the control regime underlying the application of the rules. That is, how is rule storage organized and how are the transformations triggered? The question of rule organization will be deferred to a later section but triggering issues will be considered here. In general, control regimes are some mixture of two kinds of triggering strategies: PD triggering and metaprogram controlled triggering [15, 17, 32, 34, 39]. PD triggering produces a control regime that looks like an exhaustive search process directed mostly by syntactic or semantic information local to AST subtrees. While PD regimes allow easy addition of new rules because rules can be treated as independent atoms, pure PD regimes have the problem that the triggering of the rules is based on pattern matching that is largely local to AST subtrees. This strategy leads to an overall process that is strategically blind and often induces very large search spaces.

On the other hand, the triggering choices may be made by a metaprogram that codifies some strategic goal and often employs heuristics to shortcut the search process. Metaprograms [15, 34] are algorithms and therefore, have state. This allows them to make design choices based on the earlier successes or failures. Their heuristic character, computational purposefulness and use of state information tends to reduce the search space over that of a pure PD

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<sup>4</sup> Most non-toy transformation systems use various specialized designs to attempt to overcome the inefficiencies of the “global soup” model of transformations while retaining the convenience of viewing the set of rules as a set of more or less independent transformations.

<sup>5</sup> The meta-language for definitions is: 1) **bold for code**, 2) **bold italic for meta-code**, and 3) non-bold italic underlined for comments.

search. However, the algorithmic rigidity and heuristic approach makes extensions more difficult than just dropping in a few new transformations. Finally, surprise interactions among design choices are less likely with metaprograms than with PD search because typically some combinations of design choices have been pruned away in the design of the metaprogram.

Nevertheless, both PD rules and metaprograms often lead to searches that tend to explode. Why? The short answer is *constraint propagation*, i.e., separated parts of a generated program must be coordinated in order to produce a correct program. The next section will look at this problem more closely.

### 1.3 The Constraint Propagation Problem

Constraint propagation [24] is the process whereby design choices made in one part of the generated program must be coordinated with design choices made in other parts of the program. For example, suppose that a generator is refining a container abstraction in the target program specification and the generator chooses to implement that container as a linked list. Elsewhere in the evolving program, the generator will have to choose an implementation algorithm for the container's search method. Suppose that the reusable library used by the generator allows two kinds of searches, sequential and Boyer-Moore. A Boyer-Moore search is precluded by the previous choice of a linked list implementation because for a Boyer-Moore search, the container must have the property that every element of the container can be accessed in an equal amount of time. An array has this property but a linked list does not. This constraint must be communicated between the two places in the program where these related decisions will be made.

Katz and Volper [24] show that finding a consistent set of refinements for a program specification is as hard as finding a set of values for Boolean variables that satisfy a given Boolean expression (i.e., the *Boolean Satisfiability* problem). This is the problem that theorem provers are attempting to solve. In this context, the boolean expression represents a theorem to be proved using various theorems, axioms, and a rule (or rules) of inference. The Boolean Satisfiability problem is known to be NP complete and since the Constraint Propagation problem is at least as hard as this problem, it is also NP complete.

Operationally, this means that automating the development and optimization programs is likely to face exploding spaces in the search for a consistent set of refinements and optimizations. The remainder of this paper will describe strategies that reduce this search space explosion.

## 2 Controlling Search Space Explosions

### 2.1 Overview

A central thesis of this paper is that the constraint propagation problem is best approached by solving specialized sub-problems that lend themselves to the use of specialized control regimes and metaprograms. Such strategies:

- 1) Solve narrower, more specific problems that are not NP complete (e.g., the problem of generating and integrating "implicit" target program control structures),
- 2) Use transformation control regimes that are customized to those narrower problems (e.g., break the overall translation into phases with narrow translation goals and trigger optimizing transformations based on event tags that are attached to reusable components<sup>6</sup>),
- 3) Employ domain knowledge to further prune the number of search space choices (e.g., use domain knowledge to pre-tag reusable components with names of desirable optimizations),
- 4) Limit the area of the program over which constraints must be propagated (e.g., within a single Domain Specific Language – DSL – expression), and

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<sup>6</sup> Components are definitions of domain specific classes, methods and operators. Methods and operators are implemented as transformations that are special in that they get triggered only at a predefined phase in the overall translation process. See appendix for definitions.

- 5) Organize the transformations in ways that reduce the number to be tried at any given point (e.g., group transforms in a two dimensional space that exposes only a few transforms at each point in the translation process).

Some of these strategies have been employed by existing generators (e.g., Draco [29-31] and TAMPR [12, 19] ) and some are introduced by the AOG generator. Table 1 summarizes the subproblems faced by generators as well as the strategies and mechanisms that address the subproblems. The remainder of the paper enlarges on the notions in Table 1.

**Table 1: Explosion Control Strategies and Techniques**

Source of Explosion	Explosion Control Strategy	Techniques
Numerous <b>refinements</b> and <b>constraints</b> explode derivation pathways	<b>Phased DSL to DSL Refinement</b> – Incrementally translate from higher to lower level DSLs	Mutual exclusion of DSLs produces a <b>subsetting of refinement rules</b> that hides irrelevant ones
<b>Complexity of generated code</b> explodes code reorganization choices <ul style="list-style-type: none"> <li>▪ <b>Overly complex generated code</b> explodes pattern cases</li> <li>▪ <b>Domain Specific Optimizations</b> done at code level may explode search space</li> </ul>	<b>Inter-Refinement Optimization Phases</b> – Apply specialized rules to simplify DS forms by mapping a DSL domain to itself: <ul style="list-style-type: none"> <li>▪ <b>Simplification</b> – Direct reduction of the code by removing redundancies without reorganizing the code</li> <li>▪ <b>Domain Specific Optimizations</b> done at correct domain level may use DSL knowledge to reduce search and analysis</li> </ul>	Mutual exclusion of domains implies a <b>subsetting of optimization rules</b> <p><b>Partial evaluation</b> is a metaprogram specialized to redundancy removal (e.g., <math>(X + 0) \Rightarrow X</math>)</p> <p><b>Use of domain knowledge leverages optimizations</b> (e.g., knowledge of ATN domain leverages ATN state removal optimization)</p>
<b>Implied related control</b> structures spread across DSL expressions explode the choices in integrating those controls	<b>Localization</b> – Generate customized, integrated control expressions from the implied control structures that are dispersed across DSL expressions	<b>Rule-based metaprogram specialized</b> to merge and coordinate implied controls <p><b>Speculative Refinement</b> to produce customized refinements that coordinate localization constraints across expression</p> <p><b>Explicit grouping of localization rules</b> by object (e.g., data type) and optimization goal (i.e., phase) to hide irrelevant rules</p>
<b>Global constraints</b> require coordinated global optimizations that explode constraint propagation choices	<b>Architectural Shaping</b> – Metaprogram optimizations that reshape the computation to better fit global constraints while preserving its computational function	<b>Event-triggered Tag-Directed Rules</b> preserve domain specific knowledge (via AST tags containing event-based rule invocations) for use in the code domain

### 2.1.1 Phased DSL to DSL Refinements

One way to reduce the search space is by employing **implicit** rule subsetting mechanisms to make irrelevant rules invisible in a particular situation. Systems like Draco employ distinct DSLs that can be translated in stages from high level DSLs to lower level DSLs and eventually to conventional programming languages such as C++ or Java. These distinct DSLs induce an implicit subsetting of rules that reduces the search space at each translation stage by

hiding rules not relevant to the specific DSL constructs being translated. Thus, *refinement* – the process of translating from one DSL to lower level DSLs and eventually to code – produces a series of small search spaces rather than one large search space because at each translation stage only a small number of relevant rules are available to be attempted.

### 2.1.2 Inter-Refinement Optimization Phases

At each DSL to DSL translation stage, overly complex code is often generated, which explodes the number of pattern cases that need to be used in the next translation stage. Periodic AST expression simplification is needed to prevent the rules from becoming so complex that refinement progress is impeded. Program generation often uses a kind of form *simplification* or *specialization* that basically removes redundant expressions in a DSL (e.g.,  $(X + 0) \Rightarrow X$ ) without attempting any sophisticated expression reorganization or extended inference. In AOG, this step is attempted as each new code expression is generated in an attempt to keep the expressions as simple and canonical as possible. Without this step, subsequent refinement rules become inordinately complex and thereby, explode the search space. AOG uses a *partial evaluator*<sup>7</sup> that is designed to specialize code for which some data values have become known (e.g., unrolling a loop will make loop indexes known constant values).

What is more, DSL expressions often reveal opportunities to execute certain *domain specific optimizations* that would be infeasible without the DS vantage point. [29-31] For example, an optimization rule using knowledge of an *Augmented Transition Network (ATN)* parser domain may remove an ATN state (equivalent to removing a parse rule in a conventional parser) and thereby make a significant optimization. Such an optimization would be impractical to perform once the target program is translated into a conventional programming language because the optimizer would be swamped by low-level details and low-level transformations and would no longer have the abstract, domain knowledge to guide it. The domain level knowledge provides a view of the “forest” whereas the code level provides only a view of the “trees.” Thus, DS knowledge also plays an important role by allowing *domain specific optimizations* that map from a domain to itself because the rules can use the domain knowledge to significantly improve the target computation while the program specification is still at an abstract, domain specific level. Once the abstract, DS level is translated to the conventional code, the result of the optimizations can often be seen to be quite sweeping and difficult at that level.

Thus, interlaced between each DSL to DSL refinement stage is a stage that simplifies the generated code and applies optimizations specific to the current DSL.

### 2.1.3 Localization

Domain specific languages excel at programming productivity improvements because they provide large-grain composite data structures (e.g., a graphics image) and large-grain composition operators (e.g., image addition). As a result, extensive computations can be written as APL-like one-line expressions that are equivalent to tens or hundreds of lines of code (LOC) when written in a conventional language like Java. Refining such expressions step-by-step into programming language operators and operands introduces implied control structures (e.g., loops or enumerations) for each large grain composite or large grain operator. These implied control structures are distributed (i.e., de-localized) across an expression of operators and operands. Relationships among these operators and operands invite full or partial control structure sharing across a multiple operator expression. Human programmers recognize the relation among these distributed control structures and merge them into customized control structures that minimize the redundancy of control. For example, while the controls implied by each large-grain item in a DSL expression may imply a series of passes over the data, customized control structures may be able to perform several operations on large-grain data structures in a single pass. This generation of custom control structures from the individual control elements implied by the operators and operand that are scattered across a DSL expression is called *control localization*. In AOG, control localization is automated via PD rules. A later section will follow through an extended localization example.

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<sup>7</sup> AOG's notion of partial evaluation (PE) deviates slightly from classical operational definitions but it is nevertheless performing a form of PE.

## 2.1.4 Architectural Shaping

Inter-component (i.e., cross-domain and cross-component) optimization tasks suffer several problems: 1) they are coordinating remote but related pieces of the target program, 2) they are less influenced by AST patterns than by global optimization goals, and 3) they often cannot be performed until the target program has been refined to the programming language domain by which time virtually all DS leverage is lost because the DS knowledge has been translated away. Because of these problems, conventional PD strategies for such optimization tasks may explode the search space.

These problems arise largely because the generator is trying to establish global operational properties in the target program, e.g., trying to optimize the overall code for differing machine architectures while starting out with a canonical target program specification and canonical reusable piece-parts. For example, one would like to be able to generate code optimized for a non-SIMD<sup>8</sup> architecture and with the flip of a switch generate code optimized for a SIMD architecture. Domain knowledge provides unique and valuable information about the computational goals and interrelationships of the program parts that can be used to simplify the optimization process. For example, the control structure of a convolution operator<sup>9</sup> will have an outer two-dimensional (2D) loop iterating over the image and an inner 2D loop iterating over a pixel neighborhood within that image. Further, the neighborhood computations are likely to have a special case computation method for neighborhoods that are hanging off the edge of the image. To truly exploit a SIMD machine with a parallel sum of products operator and a parallel addition operator, a human programmer would likely apply an optimization that would split the outer 2D loop into two kinds of outer loops, one to handle special case computations (e.g., neighborhood partially off the edge of the image) and one to handle the default case (e.g., neighborhood completely within the image). Such a control design will provide better pipelining of data onto the bus (i.e., no bus stalls induced by the conditional branches that check for the off-edge condition) and therefore, more optimal exploitation of the parallel operators. This “optimization goal” can be accomplished by a metaprogram whose job is to discover the pieces (i.e., the outer 2D loop associated with the convolution and the conditional test for the special case computation) and transform them into the separate control structures. In AOG, this metaprogram is a large grain transformation named **`_SplitLoopOnCases`**.

With conventional systems such optimizations are difficult because they cannot occur until the generator gets to the code level and integrates the code pieces that provide the programming details required by the loop splitting optimization. Further, the optimization (e.g., **`_SplitLoopOnCases`**) is ideally expressed in terms of abstract domain structures (e.g., the abstract form of a typical convolution and its special cases) but these structures correspond to low level programming language structures of the program that have lost any connection to that domain knowledge. That is, the generator no longer knows that a specific two-dimensional (2D) code level loop is a convolution and a specific if-then test in the body of that loop is a special case test associated with that convolution definition. The **`_SplitLoopOnCases`** optimization only works if the generator has retained a connection between the domain abstractions in which the optimization is expressed and the programming structures into which the domain abstractions are translated. While conventional optimization algorithms can (in theory) re-discover such connections, it is computationally complex and when the interrelationships among many individual optimizations are considered, re-discovery may tend to produce a search space explosion. For example, tens or hundreds of preparatory transformations may have to be discovered and run in a specific order to enable **`_SplitLoopOnCases`** and allow it to be successful.

Ironically, the creator of these reusable components (e.g., convolution definition) has the key domain knowledge in hand at the time he puts the components into the reusable library. Further, he also knows that on a SIMD machine the **`_SplitLoopOnCases`** will be a good transformation to try on these components. But conventional transformation systems allow no easy way to express early and then later exploit such information about desired optimizations.

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<sup>8</sup>SIMD is defined as Single Instruction stream, Multiple Data stream architecture.

<sup>9</sup>A convolution is the sum of the products of the pixels in a pixel neighborhood and the weights associated with the pixel positions in that neighborhood, where a neighborhood of an image is a subset of pixels centered on some pixel in the image. Neighborhoods are used to provide the specifics of various kinds of image convolution operations. Aspects of a neighborhood are defined by methods that give its size, weight values associated with various neighborhood pixel positions, special cases for dealing with the neighborhood, and so forth.

To retain such domain knowledge so that it can be directly applied once the composed components have been refined to a conventional programming language level, AOG introduces a new kind of transformation and a new control regime (i.e., tag-directed or TD Transformations<sup>10</sup>). The TD control regime preserves such early domain knowledge by adding tags to the reusable components. These tags provide direct information as to which TD transformation to invoke (i.e., the tag contains an explicit call), when to invoke it (i.e., TD-transformations are triggered by generator events) and where to focus its activity (i.e., on the component to which it is attached). This helps to avoid using complex algorithms to (partially) infer lost domain knowledge and discover the sequence of needed preparatory transformations, which in turn helps to eliminate the search spaces that can arise from the interactions of many such individual optimizations.

The remainder of the paper will examine these strategies with emphasis on those that are particular to AOG.

## 3 Localization

### 3.1 The Problem

DSLs significantly improve program productivity because they deal with large-grain data structures and large-grain operators and thereby allow a programmer to say a lot (i.e., express a lengthy computation) with a few symbols. Large-grain data structures (e.g., images, matrices, arrays, structs, strings, sets, etc.) can be decomposed into finer and finer grain data structures until one reaches data structures that are atomic with respect to some conventional programming language (e.g., field, integer, real, character, etc.). Thus, operators on large-grain data structures imply some kind of extended control structure such as a loop, a sequence of statements, a recursive function call, and so forth. As one composes large-grain operators and operands together into longer expressions, each subexpression implies not only some computation (e.g., pixel addition) that will eventually be expressed in terms of atomic operators (e.g., modular integer addition), but it also implies some control structure to sequence through those computations. Those implied control structures are typically distributed (i.e., de-localized) across the whole expression.

For example, if one defines an addition operator for images in some graphics domain and if  $\mathbf{a}$  and  $\mathbf{b}$  are defined to be graphic images, the expression  $(\mathbf{a} + \mathbf{b})$  will perform a pixel-by-pixel addition of the images. To keep the example simple and limit the number of definitions that must be introduced, suppose that the pixels are integers (i.e.,  $\mathbf{a}$  and  $\mathbf{b}$  are grayscale images) and  $\mathbf{a}$  and  $\mathbf{b}$  are the same size. Then the expression  $(\mathbf{a} + \mathbf{b})$  implies a 2D loop over  $\mathbf{a}$  and  $\mathbf{b}$ . Squaring each pixel in resulting image (represented as  $(\mathbf{a} + \mathbf{b})^2$ ) implies a second outer 2D loop. Human programmers easily identify this case as one that can be dealt with in a single 2D pass over the image.

That kind of transformation seems simple enough but the real world is much more complex and when all of the cases and combinations are dealt with, it may require tricks to avoid the search space becoming intractably large. More complex operators hint at some of this complexity. For example, consider a *convolution operator*<sup>11</sup>  $\oplus$ , which, for each pixel  $\mathbf{a}_{i,j}$  in some image  $\mathbf{a}$ , performs a sum of products of all the pixels in a neighborhood of that pixel times weights associated with the pixel positions of that neighborhood. The weights are defined separately from  $\oplus$ . Suppose the weights are defined by a domain object  $\mathbf{s}$  that is called a *neighborhood* of a pixel, where the actual pixel position defining the center of the image neighborhood will be a parameter of  $\mathbf{s}$ . Then  $(\mathbf{a} \oplus \mathbf{s})$  would define a sum of products operation for each neighborhood around each pixel in  $\mathbf{a}$  where the details of the neighborhood would come from  $\mathbf{s}$ . Thus,  $\mathbf{s}$  will contribute (among other data) the neighborhood size and the definition of the method for computing the weights. The  $\oplus$  operator definition will contribute the control loop and the specification of the centering pixel that is to be the parameter of  $\mathbf{s}$ . The translation rules not only have to introduce and merge the control structures, they have to weave together (in a consistent manner) the implied connections among the loop control, the definition of  $\oplus$  and the definition of  $\mathbf{s}$ .

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<sup>10</sup> Patent Number 6,314,562.

<sup>11</sup> A more formal definition of the convolution operator is given in the next section.

Thus, localization can be fairly complex because it is coordinating the multi-way integration of specific information from several large-grain components. The greater the factorization of the operators and operands (i.e., the separation of parts that must be integrated), the more numerous and complex are the rules required to perform the (re-) localization. As a consequence, localization has the potential to explode the solution search space. To thwart this explosion, AOG groups localization rules in special ways and makes use of domain specific knowledge to limit the explosion of choices during the localization process. Both of these strategies reduce the search space.

While this paper will focus on the Image Algebra domain [33], the de-localization problem is universal over all complex domains with large-grain operators and operands. Localization is required when the domain's language factors and compartmentalizes partial definitions of large-grain operators and data structures and then allows compositional expressions over those same operators and data structures. Other domains that exhibit DSL induced de-localization are: 1) the User Interface domain, 2) the network protocol domain, 3) various middleware domains (e.g., transaction monitors), and so forth.

### 3.2 An Example Mini-Domain

To provide a concrete context for discussing the issues of localization, this section will define a tiny portion of the Image Algebra (IA) as a mini-DSL for writing program specifications.

Domain Entity	Description	Definition	Comments
<b>Image</b>	An composite data structure in the form of a matrix with pixels as elements	$\mathbf{a} = \{ \mathbf{a}_{i,j} : \mathbf{a}_{i,j} \text{ is a pixel} \}$ where $\mathbf{a}$ is a matrix of shape $[[\mathbf{imin} : \mathbf{imax}], [\mathbf{jmin} : \mathbf{jmax}]]$	Subclasses include images with grayscale or color pixels. To simplify the discussion, assume all images have the same size.
<b>Neighborhood<sup>12</sup></b>	A matrix template overlaying a region of an image and centered on an image pixel such that the matrix associates a numerical weight with each overlay position	A neighborhood $\mathbf{s}$ is defined by a set of methods. For example, its weights are defined by a method $\mathbf{w.s}$ that computes elements of the set $\mathbf{w}(\mathbf{s}_{a[i,j]}) = \{ \mathbf{w}_{p,q} : \mathbf{w}_{p,q} \text{ is a numerical weight associated with the } [p, q] \text{ position of } \mathbf{s} \text{ centered on pixel } [i,j] \text{ of some image } \mathbf{a} \text{ where } \mathbf{s} \text{ is a neighborhood of shape } [[\mathbf{pmin} : \mathbf{pmax}], [\mathbf{qmin} : \mathbf{qmax}]] \text{ and } \mathbf{a} \text{ is an image of shape } [[\mathbf{imin} : \mathbf{imax}], [\mathbf{jmin} : \mathbf{jmax}]] \}$	Neighborhoods are objects with methods. The methods define the weights, neighborhood size, special case behaviors, and methods that compute a neighborhood position in terms of image coordinates. Notice that all methods (e.g., $\mathbf{w.s}$ ) may depend upon the image size and shape, neighborhood size and shape as well as the position of the neighborhood in the image.
<b>Convolution</b>	The convolution $(\mathbf{a} \oplus \mathbf{s})$ applies the neighborhood $\mathbf{s}$ to each pixel in $\mathbf{a}$ to produce a new image	$(\mathbf{a} \oplus \mathbf{s}) = \{ \forall i,j (\mathbf{b}_{i,j} : \mathbf{b}_{i,j} = (\sum_{p,q} (\mathbf{w}_{p,q} * \mathbf{a}_{i+p,j+q})) \}$ where $\mathbf{w}_{p,q} \in \mathbf{w}(\mathbf{s}_{a[i,j]})$ , $\mathbf{p}$ and $\mathbf{q}$ range over the neighborhood $\mathbf{s}$ ; $\mathbf{i}$ and $\mathbf{j}$ range over the images $\mathbf{a}$ and $\mathbf{b}$ )	Variants of the convolution operator are produced by replacing the $\sum_{p,q}$ operation with $\prod_{p,q}$ , $\mathbf{Min}_{p,q}$ , $\mathbf{Max}_{p,q}$ , and others; and the $+$ operation with $*$ , $\mathbf{max}$ , $\mathbf{min}$ and others.
<b>Matrix Operators</b>	$(\mathbf{a}+\mathbf{b})$ , $(\mathbf{a}-\mathbf{b})$ , $(\mathbf{k}*\mathbf{a})$ , $\mathbf{a}^n$ , $\sqrt{\mathbf{a}}$ where $\mathbf{a}$ & $\mathbf{b}$ are images, $\mathbf{k}$ & $\mathbf{n}$ are numbers	These operations on matrices have the conventional definitions, e.g., $(\mathbf{a}+\mathbf{b}) = \{ \forall i,j (\mathbf{a}_{i,j} + \mathbf{b}_{i,j}) \}$	

<sup>12</sup> To avoid unneeded complexity, this definition of Neighborhood is a slightly relaxed version of the actual IA definition.

Define the weights for concrete neighborhoods  $s$  and  $sp$  to be 0 if the neighborhood is hanging off the edge of the image, or to be

$$w(s) = P \begin{matrix} & \overbrace{\begin{matrix} -1 & 0 & 1 \end{matrix}}^Q \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \end{matrix} \quad w(sp) = P \begin{matrix} & \overbrace{\begin{matrix} -1 & 0 & 1 \end{matrix}}^Q \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \end{matrix}$$

if it is not. Given these definitions, one can write an expression for a Sobel edge detection method [33] that has the following form:

$$\mathbf{b} = [(\mathbf{a} \oplus \mathbf{s})^2 + (\mathbf{a} \oplus \mathbf{sp})^2]^{1/2}$$

This expression de-localizes loop controls and spreads them over the expression in the sense that each individual operator introduces a loop over some image(s), e.g., over the image  $\mathbf{a}$  or the intermediate image  $(\mathbf{a} \oplus \mathbf{s})$ . Consider what control structures are implicit in this expression, how they are related and how these separate implicit control structures can be woven into a minimal combined control structure.

### 3.3 A Localization Example

Implicit in the definitions of the operator and operands are the following control information and relationships<sup>13</sup>:

- 1) The instances of  $\mathbf{a}$  imply 2D loop controls that iterate through the pixels of  $\mathbf{a}$  - e.g.,  $(\forall_{\mathbf{i}, \mathbf{j}}: \mathbf{a}_{\mathbf{i}, \mathbf{j}})$  and  $(\forall_{\mathbf{v}, \mathbf{z}}: \mathbf{a}_{\mathbf{v}, \mathbf{z}})$  - where the generator creates the index variables  $\mathbf{i}, \mathbf{j}$  and  $\mathbf{v}, \mathbf{z}$ , and  $\mathbf{a}$  supplies information about the ranges of the index variables;
- 2) The convolution expressions  $(\mathbf{a} \oplus \mathbf{s})$  and  $(\mathbf{a} \oplus \mathbf{sp})$  imply two 2D loop controls that must be identical to the  $(\forall_{\mathbf{i}, \mathbf{j}}: \mathbf{a}_{\mathbf{i}, \mathbf{j}})$  and  $(\forall_{\mathbf{v}, \mathbf{z}}: \mathbf{a}_{\mathbf{v}, \mathbf{z}})$  loop controls;
- 3) The convolution expressions also imply 2D loops over the neighborhoods (e.g.,  $(\sum_{\mathbf{p}, \mathbf{q}}: (\mathbf{w} \cdot \mathbf{s} (\mathbf{a}[\mathbf{i}, \mathbf{j}], \mathbf{m}, \mathbf{n}, \mathbf{p}, \mathbf{q}] * \mathbf{a}_{\mathbf{i}+\mathbf{p}, \mathbf{j}+\mathbf{q}}))$ ) that are nested within the  $\mathbf{i}, \mathbf{j}$  and  $\mathbf{v}, \mathbf{z}$  loops, where  $\mathbf{s}$  and  $\mathbf{sp}$  supply the ranges of  $\mathbf{p}$  and  $\mathbf{q}$ , and the neighborhood weight method (e.g.,  $\mathbf{w} \cdot \mathbf{s} (\mathbf{a}[\mathbf{i}, \mathbf{j}], \mathbf{m}, \mathbf{n}, \mathbf{p}, \mathbf{q})$ ) computes the weight at the  $\mathbf{p}, \mathbf{q}$  offset of neighborhood  $\mathbf{s}$ , where  $\mathbf{s}$  is positioned at pixel  $\mathbf{i}, \mathbf{j}$  in the  $\mathbf{m}$  by  $\mathbf{n}$  image  $\mathbf{a}$ );
- 4) The instance of  $\mathbf{b}$  implies a 2D loop with generated index variables  $\mathbf{d}, \mathbf{e}$ , i.e.,  $(\forall_{\mathbf{d}, \mathbf{e}}: \mathbf{b}_{\mathbf{d}, \mathbf{e}})$  but the generator will have to infer the relationship between this loop and the other loops; and
- 5) The generator will have to infer that the 2D loop controls implied by the image instances may be merged with 2D loop controls implied by the convolution, square, plus, square root, and assignment operators based on the semantics of those operators and their operands. Operationally, this means that some of the generated index variables will be discarded and others used in their place.

Now, consider an *idealized* example of AST rewrites that will perform localization. The example will ignore many implementation complexities. Also, the AST rewrites will be re-ordered slightly to simplify the presentation. Even though the AST is a tree, the example will use a text based publication form where the tree structure is implied by the parenthetical nesting of expressions. For reference, rules will be given names. The beginning example AST is:

$$\mathbf{b} = [(\mathbf{a} \oplus \mathbf{s})^2 + (\mathbf{a} \oplus \mathbf{sp})^2]^{1/2}$$

<sup>13</sup> To keep the example simple, it will assume that  $\mathbf{a}$  and  $\mathbf{b}$  have the same dimensions.

A rule named **RefineComposite** will rewrite the AST to refine **image** instances (e.g., **b**) into **pixel** instances (e.g., **b<sub>d,e</sub>**) by introducing the control structures implied by the image instances. Three applications of the rule transforms the AST into the form:

$$(\forall_{d,e}: b_{d,e}) = [((\forall_{i,j}: a_{i,j}) \oplus s)^2 + ((\forall_{v,z}: a_{v,z}) \oplus sp)^2]^{1/2}$$

Next, the **ConvolutionOnLeaves** rule will introduce the definitions of the outer loop of  $\oplus$  and infer the equivalence of the outer loops of  $\oplus$  and the loops (i.e.,  $\forall_{i,j}$  and  $\forall_{v,z}$ ) already introduced. Because  $\oplus$  is overloaded, the new expression is using the definition of  $\oplus$  that operates on pixels whereas the previous expression was using the definition that operates on images.

$$(\forall_{d,e}: b_{d,e}) = [(\forall_{i,j}: a_{i,j} \oplus s)^2 + (\forall_{v,z}: a_{v,z} \oplus sp)^2]^{1/2}$$

Next, the **FunctionalOpsOnComposites** rule processes the square operator applied to the intermediate images (e.g., the image represented as  $(\forall_{i,j}: a_{i,j} \oplus s)$ ). Since square is a pure arithmetic function, no new loop needs to be introduced. Square can be immediately applied to each of the pixel values as they are computed by the **i,j** and **v,z** loops. Operationally, this just propagates the loops above the respective square operators.

$$(\forall_{d,e}: b_{d,e}) = [(\forall_{i,j}: (a_{i,j} \oplus s)^2) + (\forall_{v,z}: (a_{v,z} \oplus sp)^2)]^{1/2}$$

The next rewrite (the **FunctionalOpsOnParallelComposites** rule) determines that the **+** operator is adding two intermediate images whose loops can be merged. It chooses to retain the **i,j** index variables and discard the **v,z** variables. In effect, this propagates the **i,j** loop above the **+** operator and replaces the **v,z** indexes with **i,j**.

$$(\forall_{d,e}: b_{d,e}) = [\forall_{i,j}: ((a_{i,j} \oplus s)^2 + (a_{i,j} \oplus sp)^2)^{1/2}]$$

Like the square operator, the semantics of the square root operator allows the **i,j** loop to be propagated above it.

$$(\forall_{d,e}: b_{d,e}) = \forall_{i,j}: [((a_{i,j} \oplus s)^2 + (a_{i,j} \oplus sp)^2)^{1/2}]$$

Like the earlier case that combined loops over the **+** operator, the next rewrite (the **FunctionalOpsOnParallelComposites** rule) will merge the loops over the assignment operator, retaining the **i,j** index variables and discarding the **d,e** variables. The final form of loop localization is:

$$\forall_{i,j}: [ b_{i,j} = ((a_{i,j} \oplus s)^2 + (a_{i,j} \oplus sp)^2)^{1/2} ]$$

A following section will exhibit the form of the **RefineComposite** rule used in this example, but first, the form and storage organization of the PD transformation rules must be explained.

### 3.3.1 Defusing Search Space Explosions

As discussed earlier, AOG avoids NP complete approaches to program generation by solving narrower, more specialized problems with methods that are polynomial in some aspect of the program (e.g., number of nodes in an expression tree). Localization is one such specialized solution. While this narrowing of the problem is by far the most important technique for defusing search space explosions, AOG uses two additional tricks to reduce search space explosion: 1) It groups the localization rules in ways that make irrelevant rules invisible, and 2) It uses domain knowledge (e.g., knowledge about the general design of the code to be generated) to further prune the search space.

The discussion will focus on item 1 and defer discussion of item 2. Grouping transformations so that at each decision point only a small number of relevant transformations need to be tried is a good way to reduce the search space. AOG implements this idea by allowing rules to be stored under any object (e.g., a “type” object) and allows additional discrimination by further grouping the rules under an arbitrary translation *phase* name<sup>14</sup>. The phase name captures the strategic objective or job that those rules as a group are intended to accomplish (e.g., the **Localize** phase performs control localization). In addition, the object under which the rules are stored often provides some

<sup>14</sup> See appendix for example phases.

key domain knowledge that further prunes the search space. For example, in order for loop localization to move loops around, it needs to know the data flow design for the various operators. The general design of the operator's data flow is knowable by knowing the resulting type of the expression plus the details of the expression. Thus, the localization rules are stored on type objects. The individual rules determine the details of the expression (e.g., operator and operand structure) via pattern matching. As a consequence, the localization process for a specific expression of type *X* is a matter of trying all rules in the **Localize** group of the type *X* object and of types that *X* inherits from.

Operationally, AOG rules provide this organization by the rule format:

```
(⇒ XformName PhaseName ObjName Pattern RewrittenExpression Pre Post)
```

The transform's name is *XformName* (e.g., **RefineComposite**). The rule is stored as part of the *ObjName* object structure, which in the case of localization will be a type object, e.g., the **image** type. The rule is enabled only during the *PhaseName* phase, which in this context is **Localize**. *Pattern* is used to match an AST subtree and upon success, the subtree is replaced by *RewrittenExpression* instantiated with the bindings returned by the pattern match. *Pre* is the name of a routine that checks enabling conditions and performs bookkeeping chores (e.g., creating translator variables and computing equivalence classes for localization). *Post* performs various computational chores after the rewrite. *Pre* and *Post* are optional.

For example, a trivial but concrete example of a PD rule would be

```
(⇒ FoldZeroXform SomePhaseName dsnumber `( + ?x 0 ) `?x)
```

This transform is named **FoldZeroXform**, is stored on the type **dsnumber**, is enabled only in phase **SomePhaseName**, and rewrites an expression like **( + 27 0 )** to **27**. The pattern variable **?x** will match anything in the first position of expressions of the form **( + \_\_\_ 0 )**. Now, let's examine an example localization rule.

### 3.3.2 RefineComposite Rule

Among the rules used in the earlier example is **RefineComposite**, which refines an instance of a black and white image (e.g., **a**) into an instance of a black and white pixel (e.g., **a<sub>i,j</sub>**) and generates the implied control structure (e.g., **∀<sub>i,j</sub>: ...**) needed to iteratively compute the pixel values. The idealized forms shown in the example are designed for publication but gloss over some of the operational details needed for localization. In order to understand the example rule, these details must be defined. For example, each node in the AST tree has a LISP-like property list (called a *tags* list) that is used to keep translation data specific to that AST node. The tags lists are simply appended to an AST node list. For example, the expression **( + a b )** might have a tags list that contains an attribute value pair (**itype image**) indicating the type of the expression is **image**. That AST node would be represented as **( + a b (tags (itype image)) )**. All AST leaves consisting of atomic items are represented by the form **(leaf AtomicItem (tags ...))** to provide a place to hang the tags list for such nodes. Thus, the left hand side (lhs) of **RefineComposite** will have to match an AST node of the form **(leaf a (tags (itype image)))**.

By the same token, the implementation form of the transformed AST node **a<sub>i,j</sub>** will be represented for the convenience of the localization machinery. Rather than encoding **a<sub>i,j</sub>** as an inline syntactic expression that will have to be recognized and deconstructed with every use, it is represented by a translator-generated temporary symbol (e.g., **bw<sub>p27</sub>**) of type **bw<sub>pixel</sub>**. Similarly, the other translator-generated variables shown as **i** and **j** in the idealized example, will be translator-generated variables with forms more like **idx<sub>28</sub>** and **idx<sub>29</sub>**. Further, rather than encoding the loop information (e.g., **(∀<sub>i,j</sub>: ...)**) in terms of AST syntax that will have to be recognized and deconstructed by every subsequent rule, it is stored in a canonical form on the tags list, thereby allowing it to be ignored by all rules for which it is not relevant. This canonical form will be defined later.

Additionally, the **RefineComposite** rule will:

- 1) Create a record of the correspondence relationship between the large-grain composite **a** and the component **bwp27** computed from it (e.g., the expression `(_mappings (bwp27) (a))`), which will be needed to determine what can be shared between various loops and how loops nest, and
- 2) Generate a rule containing the details of the refinement relationship (e.g., a rule like `bwp27 => a[idx28, idx29]`), which will be needed to (eventually) re-express translator symbols in terms of the original data structures and the generated control variables.

How would one formulate the **RefineComposite** rule in AOG? Given a routine to generate symbols (**gensym**), a first approximation of this rule might be:

```
(=> RefineComposite Localize Image `?op (gensym `bwp))
```

But this form of the rule does not do quite enough. An image instance should be represented in the AST in the leaf form – e.g., `(leaf a ...)`. Thus, the rule will have to deal with a structure like `(leaf a (tags Prop1 Prop2 ... ))`. For robustness, the rule will allow the naked atomic image **a** as well. To accommodate this case, the rule pattern will have to use AOG’s “or” pattern operator, `$(por pat1 pat2 ...)`, which allows alternative sub-patterns (e.g., `pat1 pat2...`) to be matched.

```
(=> RefineComposite Localize Image
  `$(por (leaf ?op) ?op) (gensym `bwp))
```

Now, `(leaf a ...)` will get translated to some black and white pixel symbol such as **bwp27** with **?op** bound<sup>15</sup> to **a** (i.e., `((?op a))`). However, the rule does not yet record the relationship among the image **a**, the **bwpixel** **bwp27**, and some yet-to-be-generated index variables (e.g., **idx28** and **idx29**) that will be needed to loop over **a** to compute the various values of **bwp27**. So, the next iteration of the rule adds the name of a pre-routine (say **RCChores**) that will do the translator chores of **gensym**-ing the **bwpixel** object (**bwp27**), binding it to a new pattern variable (say **?bwp**), and while it is at it, **gensym**-ing a couple of index objects and binding them to **?idx1** and **?idx2**. The next iteration of the rule looks like:

```
(=> RefineComposite Localize Image
  `$(por (leaf ?op) ?op) `(leaf ?bwp) `RCChores)
```

Executing this rule on the AST structure `(leaf a ...)` will create the binding list `((?op a) (?bwp bwp27) (?idx1 idx28) (?idx2 idx29))` and rewrite `(leaf a ...)` to `(leaf bwp27)`. However, it does not yet record the relationship among **a**, **bwp27**, **idx28**, and **idx29**. Other references to images in the example expression will create analogous sets of **image**, **bwpixel**, and index objects, some of which will end up being redundant. In particular, new loop index variables will get generated at each image reference in the AST expression. Most of these will be redundant and other rules will be added that merge away these redundancies by discarding redundant **bwpixels** and indexes. So, the next version of the rule will create a shorthand form expressing the relationship among these items and add it to the tags list. The shorthand will have the form<sup>16</sup>

```
(_forall (idx28 idx29)
  (_suchthat (_member idx28 (_range minrow maxrow))
    (_member idx29 (_range mincol maxcol))
```

<sup>15</sup> A binding list is defined as a set of `(variable value)` pairs and is written as `((vb11 val1) (vb12 val2) ...)`. Instantiation of an expression with a binding list rewrites the expression substituting each `valn` for the corresponding `vb1n` in the expression.

<sup>16</sup> The AST is constructed using AST structures such as: `_forall` and `_sum` for expressing iterations; `_suchthat` as a holder of restrictive clauses defining the iterations; `_member` and `_range` as boolean operators used for expressing those restrictions; and `_mappings` for expressing the relationships between larger grain data structures (e.g., an image) and their smaller grain components (e.g., a black and white pixel). See appendix for a summary of the AST construction language.

```
(_mappings (bwp27) (a)))
```

The `idx` variable names will become loop control variables that will be used to iterate over the image `a` generating pixels like `bwp27`, which will eventually be refined into array references such as `(aref a idx28 idx29)`. The `_suchthat` sub-expression captures all of the relationships that will be needed to perform loop localization steps and final code generation for the loop. The `_member` clauses define the ranges of the index variables. The lists in the `_mappings` clause establish the correspondences between elements (e.g., `bwp27`, `bwp41`, etc.) and the composites from which they are derived (e.g., `a`, `b`, etc.), thereby enabling the finding and elimination of redundant elements and loop indexes.

The final form of the `RefineComposite` rule (annotated with *explanatory comments*) is:

```
(=> RefineComposite Localize Image
  `$(por (leaf ?op)      Pattern to match an image leaf structure
         ?op)           or just an image atom. Bind it to ?op
  `(leaf ?bwp          Rewrite image as the bwpixel bound to ?bwp.
    (tags              Add a property list to bwpixel structure.
      (_forall (?idx1 ?idx2) Add a loop shorthand introducing indexes,
        (_suchthat (_member ?idx1 (_range minrow maxrow)) ranges,
          (_member ?idx2 (_range mincol maxcol)) and
          (_mappings (?bwp) (?op))) DS relations.
        (itype bwpixel))) Add new type expression.
      `RCChores)         Name the pre-routine that creates
                       bwpixel & indexes.
```

Follow-on phases (`CodeGen` and `SpecRefine` respectively) will cast the resulting shorthand(s) into more conventional loop forms and refine intermediate symbols like `bwp27` into a computation expressed in terms of the source data, e.g., `a[idx32, idx33]`. But this refinement presents a constraint coordination problem to be solved. How will the later phases know to refine `bwp27` into `a[idx32, idx33]` since when it was first generated, the original relationship suggested it would be refined into `a[idx27, idx28]`? In other words, along the way, `idx27` and `idx28` became redundant and were replaced with `idx32` and `idx33`. But just replacing `bwp27` with `bwp31` in the AST at the point the redundancy is discovered does not work because the redundant indexes (e.g., `bwp27`) may occur in multiple places in the expression due to previously executed rules. Worse yet, there may be instances of `bwp27` that are yet to appear in the expression tree due to deferred rules that are pending. Other complexities arise when only the indexes are shared (e.g., between different images such as `a` and `b`). Finally, since the replacement of `bwp27` is, in theory, recursive to an indefinite depth, there may be several related abstractions simultaneously undergoing localization combination and coordination. For example, a color pixel abstraction, say `cp27`, may represent a call to the red method of the pixel class – say `(red pixel26)` – and the `pixel26` abstraction may represent an access to the image – say `a[idx64, idx65]`. Each of these abstractions can potentially change through combination during the localization process. So, how is this problem handled in AOG?

### 3.3.3 Speculative Refinements Propagate Constraints

This coordination problem is solved in AOG by the *Speculative Refinement* (SR) process, which dynamically builds refinement rules and stores them on the relevant translator generated objects, e.g., `bwp27`. In effect, the resultant set of rules propagates and coordinates constraints and translation decisions over a DSL expression. These rules are “speculative” in the sense that a rule may be altered or eliminated by subsequent localization decisions. For example, the combination process seen in the previous section incrementally modifies these rules to properly reflect the incremental removal of redundancies. Once all rules are coordinated, they will be applied in a follow-on phase called `SpecRefine`.

As an example of how SR rules are created, consider the `RefineComposite` rule shown earlier. Its pre-routine, `RCChores`, will create the several SR rules while processing various sub-expressions. Among them are:

```
(=> SpecRule89 SpecRefine bwp27 `bwp27 `(aref a idx27 idx28))
(=> SpecRule90 SpecRefine bwp31 `bwp31 `(aref a idx32 idx33))
```

Later, the pre-routine of the **FunctionalOpsOnParallelComposites** rule makes the decision to replace **bwp27** with **bwp31**. The SR rule **SpecRule89** gets changed to:

```
(=> SpecRule89 SpecRefine bwp27 `bwp27 `bwp31)
```

Thus, at the end of the loop localization phase all speculative refinement rules are coordinated to reflect the current state of localization combinations. The follow-on speculative refinement phase recursively applies any **SpecRefine** rules (e.g., **SpecRule89**) that are attached to abstractions (e.g., **bwp27**) in the AST tree. The result is a consistent and coordinated expression of references to common indexes, pixels, structure field names (e.g., **red**), and so forth.

### 3.3.4 Domain Specific Optimizations

Domain specific optimizations are opportunistic rules that run between refinement stages and attempt to improve the resultant code through use of domain knowledge. A simple example of one such DS optimization that will be triggered for the example expression is the *reduction in strength* (RIS) optimization rule<sup>17</sup> that replaces the square operation (\*\*<sup>18</sup>) with a multiplication (\*). Since the item to be squared (e.g., the expression bound to **?expr**) is an extended computation, the rule must avoid performing the computation twice. The rule's pre-routine does this by inventing a temporary variable (e.g., **t1**) to hold the value of the expression to be squared and binding it to a pattern variable (**?tmpvbl**). The transformation has the form:

```
(=> RuleName PhaseName TypeName (** ?expr 2) (* ?tmpvbl ?tmpvbl) Pre Post).
```

The pre-routine also has to generate an assignment statement of the form

```
(= ?tmpvbl ?expr)
```

to be placed into a yet-to-be-created context that dominates (i.e., precedes on all data flow paths) the statement containing the **?expr** expression. The placement of such assignment statements is handled by rules (called *deferred rules*) that the pre-routine dynamically creates. The lhs pattern of such a rule describes the expected context and the rhs rewrites the context to add the assignment statement. Deferred rules will be triggered when their target context is eventually created. The expected context for this assignment statement will be created during the **CodeGen** phase when the localization tags that evolved during the localization phase are finally converted into conventional loops. The next section describes the eventual generation of this context and the results produced by the pending deferred rules.

---

<sup>17</sup> Most compilers perform optimizations like this. However, if one waits until compile time to perform the RIS optimization, many opportunities for architectural shaping optimizations will be lost because this optimization may be a preparatory optimization that will establish some of the enabling conditions for a later TD-transform. AOG uses mostly conventional optimizations [1] but AOG's contribution is in the way in which it orchestrates a variety of complementary optimizations (some conventional, some not) to achieve a global architecture that optimizes the computation as a whole. For this example, because of the characteristics of the neighborhoods **s** and **sp** and the nature of a CPU without parallel instructions, AOG's goal is to setup the computation so that the two inner (neighborhood) loops can be unrolled and simplified into arithmetic expressions. By contrast, for a CPU with parallel instructions, AOG attempts to create an architecture that turns the inner loops into an expression that processes each neighborhood row in parallel. Section 4 treats this parallel architecture case.

<sup>18</sup> Represented as a superscript 2 in the idealized representation and as **\*\*\*** in the AST.

### 3.3.5 Localization Results

Upon completion of the speculative refinement phase, a follow-on phase (**CodeGen**) converts localization tags into AST loop forms. At this point in the generation process, the Sobel edge detection expression will be converted into the AST expression:

```
(_forall (idx32 idx33)
  (_suchthat (_member idx32 (_range 0 (- m 1)))
    (_member idx33 (_range 0 (- n 1))))
  (= (aref b idx32 idx33)
    (sqrt (+ (* t1 t1) (* t2 t2)))))
```

This form is a context that will trigger two still pending but deferred transforms that were created by the RIS optimization. They will insert the temporary variable assignment statements at the beginning of the loop body, resulting in the new form:

```
(_forall (idx32 idx33)
  (_suchthat (_member idx32 (_range 0 (- m 1)))
    (_member idx33 (_range 0 (- n 1))))
  (= t1 ( $\oplus$  (aref a idx32 idx33) s))
  (= t2 ( $\oplus$  (aref a idx32 idx33) sp))
  (= (aref b idx32 idx33)
    (sqrt (+ (* t1 t1) (* t2 t2)))))
```

Subsequent phases will further refine this form by in-lining definitions<sup>19</sup> for the convolution operator ( $\oplus$ ) inner loop, which is expressed in terms of the variables **idx32** and **idx33** as well as calls to methods of **s** and **sp** (e.g., the **w** method of **s** and **sp**). Recursive in-lining will further replace these method calls by their definitions. The inlining phase is followed by a series of phases that apply TD transformations to architecturally shape the code so that its operational behavior is better tailored to its computational environment. (See the next section.) The final code produced for a CPU without parallel instructions is:

```
for (idx32=0; idx32 < m; idx32++) /* No parallelism*/
  {im1=idx32-1; ip1= idx32+1;
  for (idx33=0; idx33 < n; idx33++)
    { if(idx32==0 || idx33==0 ||
      idx32==m-1 || idx33==n-1) /*Neighborhood Off edge?*/
      then b[idx32, idx33] = 0; /* Off edge */
      else {jm1= idx33-1; jp1 = idx33+1; /* Not off edge */
        t1 = a[im1,jm1]*(-1)+a[im1,idx33]*(-2) +
          a[im1,jp1]*(-1)+a[ip1,jm1]*1 +
          a[ip1,idx33]*2+a[ip1,jp1]*1;
        t2 = a[im1,jm1]*(-1)+a[idx32,jm1]*(-2) +
          a[ip1,jm1]*(-1)+a[im1,jp1]*1 +
          a[idx32,jp1]*2+a[ip1,jp1]*1;
        b[idx32,idx33] = sqrt(t1*t1 + t2*t2 )}}}
```

This result requires about 340 total transformations and is produced in a few tens of seconds on a 700 MHz Pentium.

<sup>19</sup> Operator and method definitions are expressed as pure functions to simplify their manipulation.

## 4 Architectural Shaping

### 4.1 Exogenous Constraints on Code Form

Up to this point, the paper has addressed the search space explosions that arise from the interactions of domain properties and refinement constraints (e.g., the refinement choice to implement a container as a linked list constrains the choices of search algorithms). These are endogenous constraints in that they arise from the essence of the computation itself and not from the nature or constraints of the external environment. Endogenous constraints act like a set of simultaneous logical equations, the solution of which is the code that achieves the DSL specification while simultaneously obeying all logical constraints. Computational correctness dictates that these constraints *must* be met. However, there are other kinds of less rigid requirements, influences and opportunities that suggest but do not require changes to the code's operational properties (e.g., the opportunity for parallel computations). Could the code be reorganized to better interact with another piece of software such as network software, middleware, user interface, data base management system, etc? Could the code be reorganized to exploit hardware parallelism? These are all "constraints" in the broadest sense and since they arise mostly because of the computational environment, they are called *exogenous constraints*. Like endogenous constraints, the exogenous constraints introduce search space explosions because there are so many alternative ways in which a computation can be reorganized and so many constraints among the individual reorganization steps. However, before examining ways to control and limit this explosion, consider the concrete result of AOG reorganizing the Sobel example to exploit hardware parallelism.

In contrast to the target code produced by AOG for a CPU without parallel instructions (above), consider how AOG alters this code to exploit parallel instructions such as the MMX instructions of the Pentium<sup>TM</sup> processor. For this case, AOG will produce code<sup>20</sup> that looks quite different:

```
{int s[(-1:1), (-1:1)]={{-1, 0, 1}, {-2, 0, 2},{-1, 0, 1}};/* MMX */
int sp [(-1:1), (-1:1)]={{-1, -2, -1}, {0, 0, 0}, {1, 2, 1}};
for (j=0; j<n; j++) b[0,j] = 0; /*Zero image edge */
for (i=0; i<m; i++) b[i,0] = 0; /*Zero image edge */
for (j=0; j<n; j++) b[(m-1),j] = 0;/*Zero image edge */
for (i=0; i<m; i++) b[i,(n-1)] = 0;/*Zero image edge */
{ for (i=1; i < (m-1); i++) /*Process inner image */
  { for (j=1; j < (n-1); j++)
    {t1 = unpackadd(padd2(padd2(pmadd3(&(a[i-1,j-1]),&(s[-1, -1])),
      pmadd3(&(a[i, j-1]),&(s[0, -1])),
      pmadd3(&(a[i+1,j-1]),&(s[ 1, -1])))),
      t2 = unpackadd(padd2 (pmadd3 (&(a[i-1, j-1]),&(sp [-1, -1])),
      pmadd3 (&(a[i+1, j-1]), &(sp [0,-1]))));
      b[i,j] = sqrt(t1*t1 + t2*t2);}}}
```

where the routines `unpackadd`, `padd2`, and `pmadd3` correspond to MMX instructions and are defined as `pmadd3 ((a0, a1, a2) , (c0, c1, c2)) = (a0*c0+a1*c1, a2*c2+0*0)`, `padd2 ((x0, x1) , (x2, x3)) = (x0+x2, x1+x3)`, and `unpackadd((x0, x1)) = (x0+x1)`. These routines lend themselves to direct translation into MMX instruction sequences. In this example, the neighborhood objects `s` and `sp` have become pure data arrays to exploit the MMX instructions. Notice that the special case that tests to see if the template is hanging over the edge of the image (i.e., "if (i==0 || j==0 || i==m-1 || j==n-1)...") has completely disappeared. Transformations have split the main loop on that test, turning the single loop of the previous version into five loops by incorporating the special case test logic into the loop control logic. Four of the loops plug zeros into the four edges of the image (i.e., the new form of the special case processing) and one loop processes the inside of the image (i.e., the non-special case processing). The fundamental difference in the derivation of the two versions

<sup>20</sup> In the name of compactness, the examples from here on will revert to the short idealized names for generated variables, e.g., `i` and `j` rather than `idx32` and `idx33`.

is in the tag-directed optimization phases. Up to that stage, the transformations that fire are the same, resulting in two interim program forms that are the same except for the tags. This version requires about 310 transformations.

## 4.2 Using Domain Knowledge in Architectural Shaping

So, how can AOG accomplish such a significant difference? The short answer is that AOG retains domain knowledge in the form of tags attached to the component parts and applies that domain knowledge by invoking the transformations named in those tags. To understand this strategy, consider the desired architecture and the domain knowledge that can be brought to bear to arrive at that architecture.

In order to exploit MMX instructions, the generated code needs to have some important architectural properties. First, the weights need to be formed into a vector to exploit the vector processing of the MMX instructions. Second, the neighborhood loop body needs to be branch free so that the vector processing instructions will not be interrupted by branch instructions.

Now, consider the domain knowledge that is available for accomplishing these architectural goals. The person who defines the neighborhood  $\mathbf{s}$  knows that it will be used in some DSL expression containing a convolution operation, for example

$$(\mathbf{t1} = (\mathbf{a} \oplus \mathbf{s})).$$

This expression will translate into some form that is conceptually equivalent to

$$\{\forall \mathbf{i}, \mathbf{j}: (\mathbf{t1}[\mathbf{i}, \mathbf{j}]: \mathbf{t1}[\mathbf{i}, \mathbf{j}] = (\sum_{\mathbf{p}, \mathbf{q}} (\mathbf{a}[\mathbf{i}+\mathbf{p}, \mathbf{j}+\mathbf{q}] * \mathbf{w.s}(\mathbf{a}[\mathbf{i}, \mathbf{j}], \mathbf{m}, \mathbf{n}, \mathbf{p}, \mathbf{q}))))\}$$

where the loop over the  $\mathbf{m}$  by  $\mathbf{n}$  image  $\mathbf{a}$  (i.e.,  $\forall \mathbf{i}, \mathbf{j}$ ) is introduced by the control localization rules, the loop over the neighborhood (i.e.,  $\sum_{\mathbf{p}, \mathbf{q}}$ ) is introduced by the definition of the  $\oplus$  operator, and the image  $\mathbf{t1}$  is introduced by the RIS optimization as a temporary holder of the computation result. When the  $\mathbf{w.s}$  (weight) method is authored, neither the  $\forall \mathbf{i}, \mathbf{j}$  loop nor the  $\sum_{\mathbf{p}, \mathbf{q}}$  loop have been generated but the author of the method knows that loops of this form will exist even though their detail structure will not be known until some specific DSL expression (e.g.,  $(\mathbf{a} \oplus \mathbf{s})$ ) has been translated. Such knowledge is highly domain specific and in the context of the  $\mathbf{w.s}$  method, it suggests how to reshape the  $\mathbf{w.s}$  definition and its future context to exploit an MMX architecture.

The body of the  $\mathbf{w.s}$  definition has the conceptual form

```

if the neighborhood is hanging off the image's edge
  then 0
  else compute w as a function of p and q

```

The off-edge test depends on the indexes  $\mathbf{i}$  and  $\mathbf{j}$  but not on  $\mathbf{p}$  or  $\mathbf{q}$  and therefore, the test should be moved outside of the  $\mathbf{p}$  and  $\mathbf{q}$  loop (by distributing the loop over the **if** statement) to establish some of the enabling conditions for a later transform that splits up the  $\mathbf{i}, \mathbf{j}$  loop to avoid bus stalls (i.e., `_SplitLoopOnCases`). The author of the method will add a tag to the **if** statement that will schedule a transformation named `_PromoteConditionAboveLoop`, which distributes the  $\mathbf{p}$  and  $\mathbf{q}$  loop over the **if** to help enable `_SplitLoopOnCases`.

Promoting the condition above the loop is motivated by the desire to eliminate the off-edge test altogether. The author knows that if the off-edge condition predicates can be incorporated into the loop control logic for the  $\mathbf{i}, \mathbf{j}$  loop, the special case logic will become a separate set of loops and the branching logic will disappear from the body of the  $\mathbf{i}, \mathbf{j}$  loop. This will eliminate branch induced bus stalls as the data flows onto the CPU bus. All of this will increase the parallelism in the computation. Thus, the author of the method also adds a tag that will trigger the `_SplitLoopOnCases` transformation. It will attempt to incorporate the off-edge condition into the loop control logic thereby forming separate loops to perform the special case computations.

Finally, the method author recognizes that the **w** values need to be formed into an array so that a whole row of neighborhood weights can be used as input to a parallel computation. Thus, the **else** branch is tagged to invoke the **\_MapToArray** transformation, which will form such an array of values and change the code using **w** accordingly. Finally, the author of the inner convolution loop (the **p, q** loop) needs to tag that loop so that it is reshaped to exploit the MMX instructions. This is accomplished by a tag on the definition of the convolution's inner loop. The tag will invoke the **\_MMXLoop** transformation to do the reshaping.

Thus, this strategy produces a set of definitions tagged with cooperating transformations that will reshape the code into an MMX form, and do so without deep analysis or search. These tags capture the domain knowledge that is available at the time the components are authored. But what about a non-MMX contexts? How does the AOG system deal with differing contexts (e.g., MMX vs non-MMX)? Simply put, it allows choice among separately tagged component versions for different contexts by using an analog of C's **ifdef**. Once the contextual constraints are chosen, the **ifdef** analog chooses the definitions that are specific to those contextual constraints. The tags on those definitions will then shape the generated code to fit the selected context.

However, one issue remains. How are the transformations invoked so that the individual steps in the reshaping process happen in the proper order? The answer is that TD tags are triggered based on events. The TD tag<sup>21</sup> format is (**\_on event TDTransformCall**). Each TD tag contains an event expression (i.e., **event**) that tells it when to trigger the call to the transformation (i.e., **TDTransformCall**). The events can be preplanned, named stages that may sequence transformations according to an abstract script. Alternatively, the events may be unscriptable opportunistic events caused by other transformation or generator actions (e.g., substitution of an expression). The next section illustrates both kinds.

### 4.3 Example Tag-Directed Transformations Exploiting Parallelism

Now, the steps in refining  $(= t1 (a[i,j] \oplus s))$ <sup>22</sup> and its role in the global optimizations will be sketched in a bit more detail. A refinement phase<sup>23</sup> named **Formals** will inline definitions for  $\oplus$  and the methods of **s** that it introduces. The definition for the MMX version of the  $\oplus$  operator with a type signature of “( $\oplus$  **image [iterator, iterator], neighborhood**)” is defined as:

```
(DefComponent24 BConvXImageArrayXNeighborhood
  ( $\oplus$  ?a[?i ?j] ?s "bind ?m & ?n to dimension fields of ?a"
    "generate ?p and ?q names from ?s")
  (_sum (?p ?q)
    (_suchthat (_member ?p (prange25 ?s ?a[?i ?j]))
      (_member ?q (qrangle ?s ?a[?i ?j]))))
    (* ?a[(row ?s ?a[?i ?j] ?p ?q)
      (col ?s ?a[?i ?j] ?p ?q)]
      (w ?s ?a[?i ?j] ?m ?n ?p ?q))
    (tags (_on (RestructLoop 1) (_MMXLoop ?p ?q))))
```

This definition is highly simplified to eliminate many of the variations and details (e.g., type checking) of the real component, thereby making it more intuitive. The parameter list of a component is a pattern and in reality, more complex than is shown here. In particular, the parameter pattern portion that binds **?m, ?n, ?p** and **?q** is expressed in the example in English without delving into the complexity of the actual pattern expression. To make the connections even more obvious, the example will use pattern variables that directly correspond to the target program variables that they will be bound to, e.g., the pattern variable **?a** will be bound to the target program variable **a**.

<sup>21</sup> See appendix for definitions of TD tags and events.

<sup>22</sup> Since  $(= t2 (a[i,j] \oplus sp))$  behaves analogously, its final form will be analogous to  $(= t2 (a[i,j] \oplus s))$  but differing in the weight constants, generated variable names and some modest structural variations due to differing partial evaluation results induced by differing constants.

<sup>23</sup> See appendix for definitions of phases.

<sup>24</sup> See appendix for definition of **Defcomponent**.

<sup>25</sup> While **prange** and **qrangle** ranges may depend upon **i, j** and properties of **a** in the general case, in this case the extra arguments are superfluous.

Further, the definition uses a bit of pseudo-code to make some of the pattern elements more intuitive (e.g., `a[i j]` for array references instead of the actual AST representation (`aref a i j`)). However, in a concession to the true AST format, the example expresses the operator and method expressions in a LISP-like, space-delimited prefix form, i.e., `(⊕ a[i j] s)` instead of `(a[i,j] ⊕ s)`.

The definition of `⊕` creates the `(?p ?q)` summation loop using methods of `?s` (i.e., `prange`, `qrange`, `w`, `row`, and `col`) to compute the neighborhood values: 1) the ranges of `?p` and `?q`, 2) the weights of neighborhood positions, and 3) the row and column positions with respect to `?a`. The `_MMXLoop` transformation will reshape the loop body into an MMX friendly form. It will be triggered by the `RestructLoop` phase event, which has an index parameter that allows other transforms with lower indexes (e.g., 0) to execute before it.

When this component is applied to `(⊕ a[i j] s)`, it will produce the binding list `((?s s) (?p p) (?q q) (?a a) (?m m) (?n n) (?i i) (?j j))` and rewrite `(⊕ a[i j] s)` as

```
(_sum (p q)
      (_suchthat (_member p (prange s a[i j]))
                 (_member q (qrange s a[i j]))))
      (*a[(row s a[i j] p q)
          (col s a[i j] p q)]
       (w s a[i j] m n p q))
      (tags (_on (RestructLoop 1) (_MMXLoop p q))))
```

The methods of `s` will be recursively inlined. Both range methods reduce to `(_range -1 1)`, the `row` method reduces to `(+ i p)` and the `col` method reduces to `(+ j q)`. The `w` method is more interesting. Its definition (slightly simplified) is:

```
(Defcomponent W (s ?a[?i ?j] ?m ?n ?p ?q)
  :pre gensignal

  (if
    (|| (== ?i 0) (== ?j 0) (== ?i (- ?m 1)) (== ?j (- ?n 1))) /*Off Edge?*/
    (then 0) /*Special Case*/
    (else /*Default Case*/
      (if (&& (!= ?p 0) (!= ?q 0))
        (then ?q)
        (else
          (if (&& (== ?p 0) (!= ?q 0)) (then (* 2 ?q)) (else 0)))
        (tags
          (_on MigrationOfMe (_MapToArray ?p ?q)
                             (_Post ?signal1)
                             (_Post ?signal2))))))
    (tags (_on ?signal1 (__PromoteConditionAboveLoop ?p ?q))
           (_on ?signal2 (_MergeCommonCondition))
           (_on (RestructLoop 0) (_SplitLoopOnCases))))
```

When this component's parameter pattern is matched against `(w s a[i j] m n p q)` from the `p` and `q` loop body, it will produce the binding list `{(?p p) (?q q) (?a a) (?m m) (?n n) (?i i) (?j j) (?signal1 signal84) (?signal2 signal85)}` where the two unique signal names are generated by the pre-routine `gensignal`. They will be used to sequence the transforms.

These signals are introduced because of a little wrinkle – an ordering dependency. `_MapToArray` must execute before `_PromoteConditionAboveLoop` lest it cause mischief to `_MapToArray`'s enabling conditions in an attempt to set up its own enabling conditions. For the same reason, the transform `_MergeCommonCondition`, which tries to establish additional enabling conditions for `_SplitLoopOnCases` (by combining common off-edge tests), must also execute after both `_MapToArray` and `_PromoteConditionAboveLoop`. To assure the

proper order of execution, the designer schedules these two transforms on the signals posted after completion of `_MapToArray` by the two `_Post` transforms.

The act of inlining the definition of `w` generates a `MigrationOfMe` event, which indicates subtree movement, for any tag in `w`'s definition subtree that is waiting on this event. This event will cause `_MapToArray` to be scheduled as the first TD transform. `_MapToArray` replaces the `if` expression to which it is attached with a reference to a newly created vector name `s[p,q]`, which it then sets about defining. First, it formulates the data declaration using the subtree to which it is attached as the body of a loop that will generate values for the newly created vector:

```
int s[(prange s a[i j]) (qrange s a[i j])] =
  (_forall (p q) (_suchthat (_member p (prange s a[i j]))
                             (_member q (qrange s a[i j]))))
    (if (&& (!= p 0) (!= q 0))
        (then q)
        (else (if (&& (== p 0) (!= q 0))
                  (then (* 2 q))
                  (else 0) ))))
```

After substitution of the definitions of `prange.s` and `qrange.s` and partial evaluation of the whole expression, it simplifies to

```
int s[(-1 1) (-1 1)]={{-1, 0, 1},{-2, 0, 2}, {-1, 0, 1}},
```

which is incorporated into a newly created, deferred dynamic transformation that will fire when the context for this declaration is eventually created. It will rewrite that context to add the declaration.

Upon completion of `_MapToArray`, the two `_Post` transformations run, posting signals `signal184` and `signal185`, which will cause scheduling of the transformations that are waiting on those signals. The `(p q)` loop now has the form

```
(_sum (p q)
  (_suchthat (_member p (_range -1 1)) (_member q (_range -1 1)))
  (* a[(+ i p),(+ j q)]
    (if (|| (== i 0) (== j 0) (== i (- m 1)) (== j (- n 1))) /*Off Edge?*/
        (then 0) /*Special Case*/
        (else s[p q]) /*Default Case*/
    (tags (_on signal184 (_PromoteConditionAboveLoop p q))
          (_on signal185 (_MergeCommonCondition))
          (_on (RestructLoop 0) (_SplitLoopOnCases))))
  (tags (_on (RestructLoop 1) (_MMXLoop p q))))
```

The `signal184` event will cause `_PromoteConditionAboveLoop` to run. In order to establish its own enabling conditions, which require the `if` statement be the first statement or operator in the body of the `(p q)` loop, it will call another transformation that distributes the `(* a[i+p,j+q]...)` expression over the `if`. This produces a `then` clause of `(* a[(+ i p) (+ j q)] 026)` which with partial evaluation becomes `0` and an `else` clause of `(* a[(+ i p) (+ j q)] s[p q])`. It next finds the `(= t1 ...)` assignment surrounding it and also distributes that over the `if`. After these transforms have run, the expression is reduced to

```
(if (|| (== i 0) (== j 0) (== i (- m 1)) (== j (- n 1))) /*Off Edge?*/
    (then (= t1 0)) /*Special Case*/
    (else /*Default Case*/
      (= t1
        (_sum (p q)
          (_suchthat (_member p (_range -1 1)) (_member q (_range -1 1))))
```

<sup>26</sup> This zero started out as a summation loop just after distribution but partial evaluation simplified it.

```

      (* a[(+ i p) (+ j q)] s[p q])
      (tags (_on (RestructLoop 1) (_MMXLoop p q))))))
(tags (_on (RestructLoop 0) (_SplitLoopOnCases))))

```

Next, **\_MergeCommonCondition** runs to establish some more enabling conditions for loop splitting. A discussion of these details is beyond the space available in this paper. Once the array has been created and the edge test promoted above the **(p q)** loop, the scheduling queue is empty, so, the next phase in the phase list – **RestructLoop** – is posted. It will trigger **\_SplitLoopOnCases**, which will restructure the **(i j)** loop that surrounds the expression shown above.

This will effect the incorporation of each of the cases of the condition test

```
(| | (== i 0) (== j 0) (== i (- m 1)) (== j (- n 1)))
```

into a separate version of the **(i j)** loop, thereby producing the five loops in the MMX code shown earlier. **\_SplitLoopOnCases** checks the enabling conditions, deconstructs both the loop control information and the branching test, and reformulates the single loop structure into five loops:

```

(_forall (i j) (_suchthat (_member i (_range 0 (- m 1)))
                          (_member j (_range 0 (- n 1)))
                          (== i 0))
          (= b[i,j] 0))
(_forall (i j) (_suchthat (_member i (_range 0 (- m 1)))
                          (_member j (_range 0 (- n 1)))
                          (== j 0))
          (= b[i,j] 0))
(_forall (i j) (_suchthat (_member i (_range 0 (- m 1)))
                          (_member j (_range 0 (- n 1)))
                          (== i (- m 1)))
          (= b[i,j] 0))
(_forall (i j) (_suchthat (_member i (_range 0 (- m 1)))
                          (_member j (_range 0 (- n 1)))
                          (== j (- n 1)))
          (= b[i,j] 0))
(_forall (i j) (_suchthat (_member i (_range 0 (- m 1)))
                          (_member j (_range 0 (- n 1)))
                          (!= i 0) (!= j 0) (!= i (- m 1))(!= j (- n 1)))
          ... default case (p q) loop ...

```

To simplify the generated control expressions, **\_SplitLoopOnCases** invokes some lightweight inference using AOG's built-in pattern language. (See the appendix for more details of the pattern language and the inference operations.) The inference step uses a set of rules that recognize the idiomatic iteration patterns associated with specific simplification strategies. For example, suppose that the control variable **i** is really a fixed constant (i.e., **(\_Suchthat (\_member i (\_range 0 (- m 1))) ... (== i 0))**). This engenders elimination of the control variable **i** from the loop control altogether and the substitution of **0** everywhere **i** appears in the loop body. That is, **(= b[i,j] 0)** would become **(= b[0,j] 0)**. The overall result is the form shown at the start of section 4.

It is important to observe that this overall shaping process is largely search and analysis free because domain specific information is used to plan the global optimizations. What transforms to run, when to run them, and what other preparatory transforms are required are all details that are mostly determined at the time that components are entered into the reusable library. Any conventional optimization methodology, would have to do a substantial amount of analysis and search to determine which transforms to run and what order to run them in.

## 4.4 Tag Logic

For a particular expression, what if the TD transforms shown in the example should fail? The short answer is that if the current tag structure will not work for a given DSL expression and set of component definitions, the definitions must be re-tagged or the DSL expression must be re-formed to allow the TD transforms to succeed. For example, convolution neighborhoods that have un-combinable case conditions may suggest reforming the DSL expression to first compute separate intermediate forms and later recombine them. The **TagLogic** phase performs this service. Tag logic is one of the directions of future research.

## 5 Contributions

Control localization contributes a generalized framework for integrating and optimizing implicit, delocalized control structures. This generalizes the loop and language specific techniques of APL [20] in that it is user extensible (e.g., to new domains) and can coordinate a range of different kinds of interdependent controls.

Speculative Refinement contributes a way to incrementally propagate constraints and optimization decisions over a DSL expression via the dynamic creation of a custom set of refinement transforms for that expression.

Dynamically created, deferred transformations contribute a general method for putting generated code into contexts that have yet to be generated.

Organizing transformations into a two dimensional memory space (*object* and *phase*) reduces the number of transformations that have to be tried for each AST subtree to a small number. For the IA domain, it is generally zero, one or two. The maximum in that domain is less than ten. Allowing variation in the kind of object that rules are stored under (e.g., a translator generated symbol) opens the door for strategies such as Speculative Refinement.

TD transformations preserve and exploit domain knowledge (e.g., the existence and relationships between image loops and neighborhood loops) in the form of tags that will orchestrate cooperating transformations, which taken as a whole, will derive desired global architectural properties. This technique eliminates most of the computation required by more conventional optimization strategies that must discover what transformations are possible, what ordering constraints exist among them and what preparatory transformations are required. TD transformations allow a search-free, distributed optimization plan to be laid out mostly in advance. The plan exploits all available knowledge (including domain knowledge) to reduce analysis and eliminate search.

AOG does not invent new transformations. Its contribution is in the search-free reuse, assembly and orchestration of well-known transformations to achieve desirable global architectures for specific computational environments.

## 6 Related Research

Good general sources for some topics in this paper include the following: generative programming [15]; transformations and meta-programming [15, 17, 32, 34, 39]; pattern matching [21, 38]; and LISP, CLOS and Prolog [13, 21, 22, 25, 28, 37].

AOG bears the strongest relation to and uses several ideas from Neighbors work. [29-31] The main differences are 1) AOG's use of metaprograms aimed at narrowly specific generation problems (e.g., localization), 2) the fact that the AOG pattern-directed transformations are organized into a two-dimensional space of *object* and *phase*, which determines which transformations are candidates for execution (i.e., are visible), 3) the control regime that provides scripts of explicitly named phases each of which defines a narrow translation job, and 4) the tag-directed control regime for architectural shaping optimizations.

The work bears a strong conceptual relationship to Kiczales' Aspect Oriented Programming (AOP) but the translation machinery appears to be quite different. [16, 26] AOP's translation mechanism seems not to be

distributed over the AST nor does it have a tag-directed control regime. In contrast, the AOG's tags are distributed over the program, are triggered by events, and may undergo transformations as the generator reasons about the domain, the program, and the optimization tags.

This work is largely orthogonal but complementary to the work of Batory. [2, 5, 15] Batory optimizes type equations to choose components from which to assemble custom classes and methods. AOG inlines and interweaves the bodies of methods invoked by compositions of method calls (i.e., DSL expressions). Thus, Batory's generation focus is at the class creation level and AOG's is at the instance application level.

AOG and Doug Smith's work are similar in that they make heavy use of domain specific information in the course of generation. [35, 15] They differ in the machinery used. Smith's work relies more heavily on inference machinery than does AOG. The reasoning that AOG does is narrowly purposeful and is a somewhat rare event (e.g., the transformation that splits the loop in the MMX example does highly specialized reasoning about loop limits). However, partial evaluation (a form of inference) [15, 39] is heavily used in AOG, which is how three level if-then-else expressions (which are interweavings of several neighborhood and operator definitions) get simplified to expressions like "`a[im1, j]*(-2)`".

The organization of the transformations into goal driven stages is conceptually similar to the work of Boyle, et al [12, 19]. However, Boyle's phases are implicit and built into the transformation metaprogram. By contrast, AOG uses lists of explicitly named phases that act like scripts and can be altered or extended by the user for different contexts. Further, the AOG work differs in that it uses domain specific information to associate TD transformations tags with reusable components as early in their life as possible to eliminate search for transformations during the reshaping phases.

The pattern, rule and traversal machinery is conceptually very similar to that of Stratego [38]. Stratego is a domain-specific language for the specification of transformation systems. Like AOG, it contains a mechanism for specification of patterns, rewrite rules, and traversal strategies, and separates the specification of traversal strategies from the specification of the rules applied by those strategies. A key difference between Stratego and AOG is in the rule grouping mechanisms. Stratego groups rules by allowing a single group name. AOG groups rules in two dimensions – *object* and *phase* – and allows rule inheritance up the class hierarchy. Additionally, new rules, phases and script-like lists of phases can be created at run time, thereby providing an avenue for programmatic redefinition of refinements and traversal processing. This produces a second order effect in the sense that early phases can dynamically create new rules and scripts for later phases that are custom tailored to the current DSL specification. Finally, Stratego does not use TD control.

The pattern language is also similar to the work of Wile [41, 42] and Crew [14]. Popart leans more toward an architecture driven by compiling and parsing notions. As such, it is influenced less by logic programming. On the other hand, ASTLOG is more similar to the AOG pattern language in that it is influenced by logic programming [13, 28]. However, ASTLOG's architecture is driven by program analysis objectives. It is a batch-oriented model that operates on a set of object files created by compile and link operations. Such a model is not well suited to dynamic manipulation and change of the AST under the control of a transformation-based generator. In addition, AOG's pattern language is distinguished from both ASTLOG and classic Prolog [13, 28] in that it does *mostly local reasoning*. That is, rather than operating on a large global data base all of which is always accessible, AOG's "data base" (or focus) is some specified locale in the AST.

There are a variety of other connections that are beyond the space limitations of the paper. For example, there are relations to other generators like SciComp's [23], Intentional Programming [15], metaprogramming and reflection [15, 34], formal synthesis systems (e.g., Specware) [15, 36], deforestation [40], transformation replay [3], the connection of goals and strategies to transformations [18] and other procedural transformation systems [27] (e.g., Refine and Reasoning5). The differences are greater or lesser across this group and broad generalization is hard. However, the most obvious broad difference between AOG and most of these systems is AOG's use of tag-directed transformations, which operate in the optimization domain and are triggered based on optimization-specific events. This makes the AOG control structure unusual, allowing planned, key optimizations to be attached to the reusable components they will optimize. Their effect is interleaved with opportunistic optimizations and partial evaluation simplifications. The overall optimization process behaves like an abstract algorithm where the algorithmic steps are phases and where the details of the steps (i.e., what operations are performed, what part of the AST they affect, and

when they get called) are partly determined by tags on the AST itself. From a different point of view, the event driven transforms behave like interrupts that allow for operations whose invocation order cannot be planned in advance and whose effect is largely reorganization, architectural shaping, and simplification.

## 7 Conclusions

AOG is being developed to study of the effects of new generator architectures on programming leverage, variability, performance, and search space size. While still early, it has demonstrated that some operators and types can be deeply factored to allow highly varied re-compositions while simultaneously allowing the generation of high performance code without huge search spaces.

## 8 References

1. Bacon, David F., Graham, Susan L., and Sharp, Oliver J.: Compiler Transformations for High-Performance Computing, ACM Surveys, Vol. 26, No. 4, (December, 1994)
2. Batory, Don, Singhal, Vivek, Sirkin, Marty, and Thomas, Jeff: Scalable Software Libraries. Symposium on the Foundations of Software Engineering. Los Angeles, California (1993)
3. Baxter, I. D.: Design Maintenance Systems. Communications of the ACM, Vol. 55, No. 4 (1992) 73-89
4. Biggerstaff, Ted J.: Anticipatory Optimization in Domain Specific Translation, International Conference on Software Reuse (1998a)
5. Biggerstaff, Ted J.: A Perspective of Generative Reuse, Annals of Software Engineering, Baltzer Science Publishers, AE Bussum, The Netherlands (1998b)
6. Biggerstaff, Ted J.: Composite Folding in Anticipatory Optimization, Microsoft Research Technical Report, MSR-TR-98-22 (1998c)
7. Biggerstaff, Ted J.: Anticipatory Optimization in Domain Specific Translation. International Conference on Software Reuse, Victoria, B. C., Canada (1998d) 124-133
8. Biggerstaff, Ted J.: Pattern Matching for Program Generation: A User Manual. Microsoft Research Technical Report MSR-TR-98-55 (1998e)
9. Biggerstaff, Ted J.: Fixing Some Transformation Problems. Automated Software Engineering Conference, Cocoa Beach, Florida (1999)
10. Biggerstaff, Ted J.: A New Control Structure for Transformation-Based Generators. In: Software Reuse: Advances in Software Reusability, Vienna, Austria, Springer (June, 2000)
11. Biggerstaff, Ted J.: Control Localization in Domain Specific Translation, International Conference on Software Reuse (2002)
12. Boyle, James M.: Abstract Programming and Program Transformation—An Approach to Reusing Programs. In: Biggerstaff, Ted and Perlis, Alan (eds.): Software Reusability, Addison-Wesley/ACM Press (1989) 361-413
13. Clocksin, W. F. and Mellish, C. S.: Programming in Prolog, Springer-Verlag (1987)
14. Crew, R. F.: ASTLOG: A Language for Examining Abstract Syntax Trees. Proceedings of the USENIX Conference on Domain-Specific Languages, Santa Barbara, California (1997)
15. Czarnecki, Krzysztof and Eisenecker, Ulrich W.: Generative Programming: Methods, Tools, and Applications. Addison-Wesley (2000)
16. Elrad, Tzilla, Filman, Robert E., Bader, Atef (Eds.): Special Issue on Aspect-Oriented Programming. Communications of the ACM, Vol. 44, No. 10 (2001) 28-97
17. Feather, Martin: A Survey and Classification of some Program Transformation Approaches and Techniques, In Program Specification and Transformation, Elsevier (North-Holland), IFIP (1987)
18. Fickas, Stephen F.: Automating the Transformational Development of Software, IEEE Transactions on Software Engineering, SE-11 (11), pp 1286-1277, (Nov. 1985)
19. Fitzpatrick, Stephen, Harmer, Terence J., Stewart, Alan, Clint, Maurice and Boyle, James M.: The Automated Transformation of Abstract Specifications of Numerical Algorithms into Efficient Array Processor Implementations, Science of Computer Programming Vol. 28, No. 1 (1997) 1-41
20. Guibas, L.J. and D.K. Wyatt: Compilation and Delayed Evaluation in APL, Fifth Annual ACM Symposium Principles of Programming Languages, (1978) 1-8
21. Graham, Paul: On Lisp: Advanced Techniques for Common Lisp, Prentice-Hall (1994)
22. Graham, Paul: The ANSI Common Lisp, Prentice-Hall (1996)
23. Kant, Elaine: Synthesis of Mathematical Modeling Software, IEEE Software, (May, 1993)
24. Katz, M. D. and Volper, D.: Constraint Propagation in Software Libraries of Transformation Systems, International Journal of Software Engineering and Knowledge Engineering, 2, 3 (1992)

25. Keene, Sonya E.: Object Oriented Programming in Common Lisp: A Programmers Guide to the Common Lisp Object System, Addison-Wesley (1989)
26. Kiczales, Gregor, Lamping, John, Mendhekar, Anurag, Maede, Chris, Lopes, Cristina, Loingtier, Jean-Marc and Irwin, John: Aspect Oriented Programming. Tech. Report SPL97-08 P9710042, Xerox PARC (1997)
27. Kotik, Gordon B., Rockmore, A. Joseph, and Smith, Douglas R.: Use of Refine for Knowledge-Based Software Development, Western Conference on Knowledge-Based Engineering and Expert Systems (1986)
28. Malpas, John: Prolog: A Relational Language and it Applications, Prentice-Hall (1987)
29. Neighbors, James M.: Software Construction Using Components. PhD Thesis, University of California at Irvine, (1980)
30. Neighbors, James M.: The Draco Approach to Constructing Software From Reusable Components. IEEE Transactions on Software Engineering, SE-10 (5), pp 564-573, (Sept. 1984)
31. Neighbors, James M.: Draco: A Method for Engineering Reusable Software Systems. In: Biggerstaff, Ted and Perlis, Alan (eds.): Software Reusability, Addison-Wesley/ACM Press (1989) 295-319
32. Parsch, Helmut A.: Specification and Transformation of Programs, Springer-Verlag (1990)
33. Ritter, Gerhard X. and Wilson, Joseph N.: Handbook of Computer Vision Algorithms in the Image Algebra, CRC Press, (1996)
34. Sheard, Tim: Accomplishments and Research Challenges in Meta-Programming, SAIG 2001 Workshop, Florence, Italy (Sept., 2001)
35. Smith, Douglas R.: KIDS-A Knowledge-Based Software Development System. In: Lowry, M. & McCartney, R., (eds.): Automating Software Design, AAAI/MIT Press (1991) 483-514
36. Srinivas, Y. V.: Refinement of Parameterized Algebraic Specifications. In: Bird, R. and Meertens, L. (eds.): Proceedings of a Workshop on Algorithmic Languages and Calculi. Alsac FR. Chapman and Hill. (1997) 164-186
37. Steele Jr., Guy L., Common Lisp: The Language (Second Edition), Digital Press (1990)
38. Visser, Eclo: Strategic Pattern Matching. In: Rewriting Techniques and Applications (RTA '99), Trento, Italy. Springer-Verlag (July, 1999)
39. Visser, Eclo: A Survey of Strategies in Program Transformation Systems. In: B. Gramlich and S. L. Alba, editors, Workshop on Reduction Strategies in Rewriting and Programming (WRS '01), Utrecht, The Netherlands (May 2001)
40. Wadler, Philip: Deforestation: Transforming Programs to Eliminate Trees. Journal of Theoretical Computer Science, Vol. 73 (1990) 231-248
41. Wile, David S.: Popart: Producer of Parsers and Related Tools. USC/Information Sciences Institute Technical Report, Marina del Rey, California (1994) (<http://www.isi.edu/software-sciences/wile/Popart/popart.html>)
42. Wile, David S.: Toward a Calculus for Abstract Syntax Trees. In: Bird, R. and Meertens, L. (eds.): Proceedings of a Workshop on Algorithmic Languages and Calculi. Alsac FR. Chapman and Hill (1997) 324-352

## 9 Appendix: Elements of AOG

### 9.1 Reusable Components

Component	Form	Description
DSL Class Definition	<pre>(defAOGClass Name (Super)   Form   (SlotName1 InitVal1)   (SlotName2 InitVal2)...) </pre>	This creates an CLOS class named <i>Name</i> whose superclass is <i>Super</i> . It has slots <i>SlotName1</i> , <i>SlotName2</i> , and so forth. The slots may optionally have initial values. <i>Form</i> will create an instance of the class.
Pattern-Directed Transformation	<pre>(=&gt; XformName Phase   XFormGroup Pattern   Rewrite [Pre] [Post]) </pre>	The transform's name is <i>XformName</i> and it is stored as part of the <i>XFormGroup</i> object structure (which might be a type or other specific kind of grouping object, for example). It is enabled only during the <i>Phase</i> phase. <i>Pattern</i> is used to match an AST subtree and upon success, the subtree is replaced by <i>Rewrite</i> instantiated with the bindings derived during the pattern match. <i>Pre</i> is the name of a routine that checks further enabling conditions and may perform bookkeeping chores (e.g., creating translator variables). <i>Post</i> performs chores after the rewrite. <i>Pre</i> and <i>Post</i> are optional.
DSL Operator Definition	<pre>(DefComponent CompName   (Operator .   ParameterPat)   [Pre: PreName]   [Post: PostName] Body) </pre>	Equivalent to <pre>(P CompName Formals TypeofOperator   EnhancedPattern Body PreName   PostName)</pre> <p>where <i>TypeofOperator</i> is the type of <i>Operator</i>, <i>Formals</i> is the phase where operator and method inlining occurs, and <i>EnhancedPattern</i> is a pattern automatically derived from <i>(Operator . ParameterPat)</i> enhanced with hidden implementation machinery that simplifies the writing of operator definitions.</p>
DSL Class Method	<pre>(DefComponent MethodName   (Object . ParameterPat)   [Pre: PreName]   [Post: PostName] Body) </pre>	Equivalent to <pre>(=&gt; MethodName Formals Object   EnhancedPattern Body PreName   PostName)</pre> <p>where <i>Formals</i> is the phase where operator and method inlining occurs, and <i>EnhancedPattern</i> is a pattern automatically derived from <i>(MethodName Object . ParameterPat)</i> with enhancements analogous to those of operator definitions.</p>

Dynamic Deferred Transform	Dynamically created as ( <b>GenDeferredXform</b> <i>XformName ContextPattern</i> <i>Rewrite Bindings</i> )	These transforms are part of specialized machinery for moving generated code to contexts that do not yet exist when the code is generated. Given initial bindings of <b>Bindings</b> , at some future time when <b>ContextPattern</b> matches some newly created context, then that context is replaced by <b>Rewrite</b> .
Type Inference Rules for Operators	( <b>DefOPInference</b> <i>OpType</i> ( <i>Operator Type1 Type2</i> ...) <i>ResultType</i> )	This macro generates a pattern that will compute a type for the <b>Operator</b> expression where the expression's parameters have types <b>Type1 Type2 ....</b> This rule is stored in the <b>OpType</b> class. In addition to simple type names, various pattern operators allow indefinite spans such as zero or more instances of types, one or more, and so forth. The <b>ResultType</b> may be either 1) an explicit type name, 2) an integer parameter position indicating that the resulting type is the same as that parameter, or 3) the keyword <b>last</b> indicating the type of the last argument. After a successful match of a type inference pattern, the binding list will contain the inferred type bound to <b>?itype</b> .
Type Inference Rules for Methods	( <b>DefMethodInference</b> <i>ObjType (MethodName</i> <i>Object Type1 Type2 ...)</i> <i>ResultType</i> )	This generates a pattern that computes the inferred type of a call to the <b>MethodName</b> method of object <b>Object</b> . The rule is stored in the <b>ObjType</b> class.

## 9.2 Inheritance Hierarchy

The inheritance hierarchy is user extensible and each newly added domain will introduce new operators, data types, method names, and perhaps other DSL classes. The inheritance hierarchy below shows only the classes necessary for discussing the graphics domain used in this paper.

- **DSTypes** – Top DS type
  - **ADT** – DSL entities that can have methods
    - **IATemplate** – General sub-image entity having size, shape, weights, and special case behaviors for use by image operators.
      - **Neighborhood** – Commonly used special case of IATemplate.
  - **DSOperands** – Data objects (both DSL and conventional programming).
    - **Composites**
      - **CompositeProgrammingDomainTypes** – Structures such as Array, Range, etc.
      - **ImageDomainTypes** – Image, Pixel, Channel, etc.
    - **FauxLispTypes** – Duplicates Lisp types & provides home for AOG data.
    - **Scalars** – AOG's atomic data types.
  - **DSOperators** – DS languages and programming operators (overload-able)
    - **ArithmeticOperators** – Both arithmetic operators (e.g., plus and minus) and functions (e.g., sin and floor)
    - **ControlOperators** – If-Then-Else, code blocks, etc.
    - **DataOperators** – Cons, list, etc.
    - **ImageOperators** – Various convolution, template, neighborhood, and pixel operators.
    - **LogicalOperators** – Operators such as and, or, not, xor.
    - **Methods** – ADT method names.
    - **RelationalOperators** – Operators such as less and greater.

### 9.3 AST Internal Representation

For the programming language domain, the AST is expressed in terms of CLOS objects that correspond to LISP types (classes), operators, methods and data (e.g., +, **lessp**, etc.). Extensions are made for uniquely non-LISP-like constructs. For DSL expressions, the AST is similarly expressed in terms of DSL specific classes, operators, methods and data (e.g., the **neighborhood** class, + operator overloaded for images, and **convolution** operator). See the inheritance hierarchy for an overview of these CLOS classes. The AST representation is user extensible so that new constructs are easily added when new metaprograms or new transformations require new specialized forms for intermediate representations. For example, control localization is simplified if the loops are expressed in higher-level canonical forms than programming language forms. This allows the abstract essence of the loop operation (e.g., a summation) to be determined without having to recognize some extended pattern of operators, types, and forms. Secondly, the transforms are simplified if the loop expressions are broadly canonical in form and express detail variations by the addition of a few stereotypical predicate expressions. The following table defines a few of the specialized AST constructs that are germane to this paper.

Constructor	Definition	Explanation
Leaf Node Constructor	<b>(leaf atom taglist)</b>	An atomic AST leaf with a property list.
Loop Forms	<b>(loopop vbls (_suchthat constraints) body)</b>	<i>loopop</i> may be one of <b>_sum</b> (or $\Sigma$ ), <b>_forall</b> (or $\forall$ ), <b>_product</b> (or $\Pi$ ), <b>_min</b> , <b>_max</b> , <b>_xormin</b> , <b>_xormax</b> , or <b>_reduce</b> ; <i>vbls</i> is a list of the loop control variables; <i>constraints</i> is a list of the predicates expressing the loop control details; and <i>body</i> is the loop body.
Constraint Predicates	<b>(cpred . arglist)</b>	<i>cpred</i> may be one of <b>_member</b> , <b>_range</b> , <b>_mappings</b> , etc. Predicate constraints provide information about variables in <i>vbls</i> such as beginning, ending and increment values or constant values. They also provide relationships among large grain components, their subcomponents and loop variables that will be required to compute the subcomponent values.

### 9.4 Tag Structures

Component	Form	Description
Tags list on an AST non-leaf subtree	<b>(ASTSubtree . (tags . taglist))</b>	An arbitrary AST subtree ( <b>ASTSubtree</b> ) may have a tags list ( <b>tags . taglist</b> ), which is the analog of a LISP property list for LISP atoms. The portion <b>taglist</b> after the key word <b>tags</b> is a simple association list. Like LISP property lists, tag lists are used for storing arbitrary information, one important example of which is a TD-transformation tag (see below).
Tags list on an AST leaf subtree	<b>(leaf LeafItem (tags . taglist))</b>	An AST leaf (i.e., a non-list subtree such as an atom) may also have a tags list by embedding the leaf value

AST leaf subtree	<i>taglist</i> )	( <i>LeafItem</i> ) in a leaf list.
General form of a tag list entry	( <i>attribute</i> . <i>valuelist</i> )	A tag is an attribute-value structure for associating arbitrary <i>attribute</i> keys with a list of values ( <i>valuelist</i> ). It is analogous to a LISP property. An example of a commonly used tag is the inferred type of the expression, i.e., ( <i>itype typename</i> ).
TD-Transformation Tag	( <i>_on event</i> <i>(TDtransform1</i> . <i>parmlist1</i> ) <i>(TDtransform2</i> . <i>parmlist2</i> ) ...)	A TD-Transform tag may trigger a single transform or a list of transforms. Transforms in a block are scheduled sequentially with each completing before the next is scheduled. When <i>event</i> is posted, each ( <i>TDtransformN subtree root mytags</i> . <i>parmlistN</i> ) is scheduled for execution, where three implementation variables are introduced by the scheduler. <i>subtree</i> is the AST subtree to which the transformation tag is attached, <i>root</i> is the current DSL expression tree, and <i>mytags</i> is the tag list of <i>subtree</i> . The <i>TDtransformN</i> may have user defined parameters (i.e., <i>parmlistN</i> ) comprising <i>?variable</i> -s or constants.

## 9.5 Events

Events are used to trigger Tag-Directed transforms. Events arise in several ways: 1) the name on a script of planned phase names (see next section) will be posted by the TD-transformation scheduler whenever its scheduling queue becomes empty; 2) a TD-transformation may post an event in the course of its operation (e.g., when it substitutes a new structure that has a TD-transform in its tags list waiting for the event **SubstitutionOfMe**); 3) a transformation may generate a signal name and bind it to a specific global variable (e.g., **?signal**); or 4) some of the AOG kernel software such as the Partial Evaluator may generate an opportunistic signal particular to their operation (e.g., **SimplificationOfMe** would be generated on the simplification of an AST expression, but only if that expression was waiting on that event).

Event Name	Description
<i>eventname</i>	Any arbitrary <i>eventname</i> can be posted as an event and will trigger any transformation tag waiting on that name. Planned phases are initiated by simply posting an event whose value is the name of the phase. Except for events that are stipulated to be local (i.e., applying only to the subtree that generated the event), events are global and apply to tags anywhere in the AST expression tree.
<i>?variable</i>	The value of any <i>?variable</i> can be used as an event name. Events can be dynamically created and used as signals between cooperating transformations. Signaling is one way to implement inter-transformation dependencies.
( <i>eventname ParameterList</i> )	Events can have parameters. See <b>OnCompletionOfXform</b> below for an example.
<b>SubstitutionOfMe</b>	Local event. If the AST subtree that has a <b>SubstitutionOfMe</b> tag expression at its root is

	<p>moved into a new position in the AST or into a newly created AST subtree, the tag or tags waiting on this event are scheduled. If the subtree being substituted has a <b>SubstitutionOfMe</b> tag expression at a non-root position, this event is not posted. The event is “local” in that, it applies only to the subtree that is moved and nowhere else in the AST.</p>
<b>MigrationOfMe</b>	<p>Local event. Same as <b>SubstitutionOfMe</b> except that the tag does not have to be at the root of the moved subtree.</p>
<b>SimplificationOfMe</b>	<p>Local event. Subtree has been changed by partial evaluation.</p>
<b>(CompletionOfXform <i>XformName</i>)</b>	<p>Global event. When <b><i>XformName</i></b> completes, any transform waiting on this event will be scheduled.</p>

## 9.6 Phases

Phases are completely user definable and extendible (statically or dynamically). Further, there are no limitations on a transformation recursively introducing a list of sub-phases in the midst of a phase. While there are no built-in phases, the phases used for the domain discussed in this paper are shown below.

Phase Type	Phase	Description
<b>PD Refinements</b>	<b>TypeInfer</b>	Initial type inference over DSL expression. Thereafter, type inference is incremental.
	<b>DSLOpt</b>	Algebraic manipulation of formulas in the domain to optimize the computation.
	<b>EnableLocalize</b>	Manipulate DSL expressions to remove impediments to localization success. For example, $\mathbf{b} = [ [(\mathbf{a} \oplus \mathbf{s})^2 + (\mathbf{a} \oplus \mathbf{sp})^2]^{1/2} \oplus \mathbf{sr}]$ would be rewritten to $\{ \mathbf{t1} = [(\mathbf{a} \oplus \mathbf{s})^2 + (\mathbf{a} \oplus \mathbf{sp})^2]^{1/2} ; \mathbf{b} = (\mathbf{t1} \oplus \mathbf{sr}) \}$ so that $\mathbf{t1}$ can be computed in a single pass over $\mathbf{a}$ .
	<b>TagLogic</b>	Analyze tags with respect to the structure of the expression being translated and revise tags, if necessary. For example, neighborhoods with non-combinable special case tests may suggest separate passes over image to compute intermediate images so that each pass can fully exploit parallelism.
	<b>Localize</b>	Localize control to minimize control computation (e.g., the number of passes over an expression) and maximize sharing (e.g., share common indexes).
	<b>SpecRefine</b>	Speculative refinement maps de-localized control into shared control according to the localization decisions.
	<b>CodeGen</b>	Reformulate control structures into conventional programming language forms now that control (e.g., loop) locations are fixed.
	<b>Formals</b>	Inline definitions for operators and methods (e.g., convolutions and neighborhood methods).
<b>TD Optimizations</b>	<b>RestructLoopBody</b>	Prepare loop bodies to enable loop restructuring (e.g., distributing an <b>if</b> statement over a loop to enable loop splitting). Also, perform opportunistic optimizations (e.g., merging <b>if</b> -s with common tests).
	<b>RestructLoop</b>	Tailor looping to computational environment (e.g., loop unrolling or loop splitting).
	<b>RefineLoop</b>	Clean up optimizations (e.g., hoisting code out of loops).

## 9.7 Pattern Elements

The left hand side of transformations (or rules) are expressed in the following backtracking-based [21, 22] pattern language.

Pattern Element	Explanation
<i>Literal Data</i>	Succeeds if it matches the same literal data in the AST. The pattern language is an <i>inverse quoting</i> representation, so that literal data is represented as itself without any syntactic adornment. Non-literal data (e.g., pattern variables or operators) will have syntactic adornment such as a “?” or “\$” prefix.
<i>?vblname</i>	Pattern variable. If <i>?vblname</i> is unbound in the current match, it will match and bind to any AST subtree. If <i>?vblname</i> is bound, it will match only if its current value matches the current subtree of the AST.
<i>\$(por pattern1 pattern2 ... patternN)</i>	Succeeds if any of the patterns succeed.
<i>\$(pand pattern1 pattern2 ... patternN)</i>	Succeeds if all patterns succeed.
<i>\$(pnot pattern)</i>	Succeeds if the pattern fails.
<i>\$(none pattern1 pattern2 ... patternN)</i>	Succeeds if the current AST subtree matches none of the patterns.
<i>\$(bindconst ?variable constant)</i>	Bind the <i>constant</i> value to <i>?variable</i> and succeed, allowing the pattern match to advance without advancing the current position in the AST.
<i>\$(bindvar ?variable pattern)</i>	This will cause whatever the pattern argument matches in the current AST subtree to be bound to <i>?variable</i> . This is an analog of the SNOBOL4 “\$” operator.
<i>\$(is ?variable expression)</i>	The <i>expression</i> is evaluated by LISP and its value bound to the <i>?variable</i> .
<i>\$(ptest SingleArgumentLispFunction)</i>	Calls the LISP function using the current item in the AST as its argument. The pattern match succeeds or fails based on the result of <i>SingleArgumentLispFunction</i> . The function may be a lambda expression.
<i>\$(papply funct arg1 arg2 ... argn)</i>	Applies LISP function <i>funct</i> to the arguments without advancing the pattern matcher’s data pointer. If <i>funct</i> returns non-nil, the match succeeds. Otherwise it fails. A non-nil value may be a pair of the form ( <i>variable value</i> ) to indicate a binding pair to be added to the current binding list.
<i>\$(plisp &lt;list of Lisp statements using pattern ?variables&gt;)</i>	Executes the list of LISP statements succeeding or failing based on the return value of the Lisp code. A nil value causes failure and non-nil succeeds. A non-nil value may be a binding pair ( <i>variable, value</i> ) to be added to the binding list.
<i>\$(plet LetList Pattern)</i>	This ensures that the <i>?variables</i> mentioned in the <i>LetList</i> are local to the <i>Pattern</i> .
<i>\$(pdeclare LetList Pattern)</i>	This is a mechanism that allows a simpler interface to LISP code by mentioning <i>?variables</i> so that AOG macros can invisibly set up a LISP scope for <i>?variables</i> whose names cannot be determined at pattern definition time.
<i>\$(pmatch Pattern Data)</i>	Recursively match <i>Pattern</i> against <i>Data</i> that has been bound in the parent match. This basically makes patterns easier to read by taking sub-matching out of line.
<i>\$(psuch Slotname ?vbl Pattern)</i>	Match <i>Pattern</i> against the value of slot <i>Slotname</i> of the CLOS object bound to <i>?vbl</i> .
<i>\$(remain ?variable)</i>	Bind <i>?variable</i> to the remainder of the list from the current position in the AST. This is the analog of the SNOBOL4 rem operator.
<i>\$(spanto ?variable Pattern)</i>	Bind <i>?variable</i> to the remainder of parent list up to but not

	including that expression that matches <b>Pattern</b> .
<b>\$(oneormore Pattern)</b>	Succeeds if <b>Pattern</b> occurs one or more times at the current position in the AST tree.
<b>\$(zeroormore Pattern)</b>	Succeeds if <b>Pattern</b> occurs zero or more times at the current position in the AST tree.
<b>\$(within Pattern)</b>	Succeeds if there is an instance of <b>Pattern</b> anywhere within the current AST subtree. The search order is from the leaves up to the root of the subtree.
<b>\$(preorderwithin Pattern)</b>	Succeed if there is an instance of <b>Pattern</b> anywhere within the current AST subtree. The search order is from the root down.
<b>\$(pat ?variable)</b>	Match the current item against the pattern bound to <b>?variable</b> . This allows dynamically computed patterns. For example, the data may contain patterns that describe other portions of the data.
<b>\$(pmark)</b>	This puts a Prolog-like mark on the choices stack to indicate where the next cut will be stopped.
<b>\$(pcut)</b>	User invoked Prolog-like cut to cause search to abandon remaining choices back to the last marked choice point.
<b>\$(psucceed)</b>	Allows the user to assure success of a pattern.
<b>\$(pfail)</b>	User invoked fail causes search to backup to last choice point and select the next choice. This is often used to iterate through all possible matches.
<b>\$(ptrace LispExpression Label)</b>	Pattern debugging facility. This produces trace printout of the label if the lisp expression evaluates to true.
<b>(&lt;- consequent antecedent)</b>	Defines a Prolog-like inference rule whose <b>antecedent</b> defines a method for achieving the goal expressed by the <b>consequent</b> . This is basically a pattern that can be "called" based on its <b>consequent</b> pattern. These are the inference rules used by AOG during the generation process.
<b>\$(pprove goal)</b>	Will invoke a Prolog-like rule whose <b>consequent</b> matches <b>goal</b> . This is the mechanism by which AOG does inference.
<b>(RuleSet (RulesetName super) rule1 rule2 ... rulen)</b>	Defines a rule set <b>RulesetName</b> containing the inference rules <b>rule1 rule2 ... rulen</b> . <b>RulesetName</b> inherits rules from another rule set object <b>super</b> .
<b>\$(with-rules (RulesetName) Pattern)</b>	Defines the starting <b>RulesetName</b> for use with any <b>pprove</b> operator in pattern.
<b>\$(op pattern1 pattern2 ...)</b>	This is the general form of any arbitrary pattern operator. It is represented internally as <pre>(*PAT* (LAMBDA (*CONT* DATA BINDINGS) (FUNCALL '=op *CONT* '(Pattern1 Pattern2...) DATA BINDINGS)))</pre> where <b>=op</b> is a Lisp pattern matching routine built using continuations, <b>*CONT*</b> is a continuation, <b>DATA</b> is the AST being matched, and <b>BINDINGS</b> is the current binding list. If the user writes pattern operators according to a few simple rules, the pattern language can be user extended to include arbitrary new pattern operators.
<b>#.PatternName</b>	Shared pattern, which is the value of the LISP variable <b>PatternName</b> . This is the analog of a non-terminal symbol in a context free grammar.

## 9.8 RHS Meta-Operators

Constructor	Explanation
<b>CommaSplice</b>	Evaluated at RHS construction time, this operator splices a list value into another list. It is the analog of the LISP ,@ operator.
<b>CommaIf</b>	Meta-If allows a choice among different forms of RHS expressions to be made at RHS construction time. For example, computational architectures may indicate differing sets of initial tags on definitions.

## 9.9 Pattern Application Scope

The with-matching macro is used to provide a transparent interface between the pattern matching machinery and the transformation code that will be manipulating the AST structures discovered during the pattern matching.

<p><b>(with-matching <i>Pattern Data Bindings Body</i>)</b></p>	<p><b>with-matching</b> is a part of the pattern matching machinery used in PD and TD rules as well as AOG internal structures. It is a LISP macro that establishes a scope for pattern variables and several hidden variables. This coordinates pattern variables such as <b>?vbl</b> with corresponding LISP <b>let</b> variables and thereby, hides much of the messy implementation machinery. Additionally, it introduces and computes values for three hidden <b>let</b> variables that are part of the deep implementation machinery and are available to the body of the macro:</p> <ol style="list-style-type: none"> <li>1) <b>result</b>, which is a two-tuple containing the success/fail flag and the binding list, if any,</li> <li>2) <b>success</b>, the success/fail flag, and</li> <li>3) <b>newbindings</b>, the binding list produced by the matcher, if the match was successful.</li> </ol> <p>The pattern match is executed, the values of the hidden variables are bound to their LISP <b>let</b> variables, the pattern variable bindings are bound to their corresponding LISP <b>let</b> variables, and <b>Body</b> is eval-ed in this <b>let</b> context.</p>
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